VOLUME 26 DECEMBER, 1938 NUMBER 12

PROCEEDINGS
of
The Institute of Radio Engineers

Application Blank for Associate Membership on Page XIII

Institute of Radio Engineers Forthcoming Meetings

CINCINNATI SECTION
December 13, 1938

CLEVELAND SECTION
December 22, 1938

DETROIT SECTION
December 16, 1938

LOS ANGELES SECTION
December 20, 1938

NEW YORK MEETING
January 4, 1939

PHILADELPHIA SECTION
January 3, 1939

PITTSBURGH SECTION
December 20, 1938

WASHINGTON SECTION
December 12, 1938

OSCAR B. HANSON
Director, 1938

Oscar B. Hanson was born on February 11, 1894, in Huddersfield, England. After being privately educated in England he came to America and entered the Marconi School in 1912. On obtaining his license, he went to sea as an operator. From 1917 to 1920 he worked in the testing department of the Marconi Company, becoming chief testing engineer, and then returned to sea.

In 1921 he joined the staff of WAAM, in Newark, New Jersey, and a year later became assistant to the plant engineer of WEAF, then operated by the American Telephone and Telegraph Company. He has directed the technical operations and engineering activities of the National Broadcasting Company since its founding in 1926. He has been responsible for the design and construction of the NBC studios at 711 Fifth Avenue in New York City, in the Merchandise Mart in Chicago, in Washington, D. C.; Cleveland, Ohio; Hollywood, California; and the present Radio City studios in New York. He is in charge, also, of the television work of the National Broadcasting Company.

Mr. Hanson joined the Institute as an Associate in 1918 and transferred to Member in 1927.

INSTITUTE NEWS AND RADIO NOTES

Proceedings Format

The January, 1939, issue of the PROCEEDINGS will exhibit the first major changes in its appearance since Volume 1, Number 1, came off the presses in 1913. For a quarter of a century, its granite-gray cover and "pocket" size have been familiar to radio engineers the world over. Our library shelves carry positive evidence of the many thousands of pages which have been distributed within these gray covers. We have been able to include within their confines the most complex and lengthy mathematical formulas and when illustrations were too large, we could always paste in a folded sheet to accommodate them. With these thoughts in mind, it is very reasonable to ask "Why not let well enough alone?"—"Why make a larger page size which will be more difficult to read on a train"—"Why interrupt the smooth continuity of the volumes on our library shelves?"—"Why relinquish an appearance which is known the world over?"—"Why—?"

Several times during the past ten years, our Board of Directors has considered changing the PROCEEDINGS size, shape, or color. Invariably the answer was "No." However, in October of 1937, the Board of Editors met in response to some pointed criticisms of the methods of handling various types of material published regularly in the magazine and the rate at which papers were published. A change in size offered the most effective attack on these problems but it was very evident that there was little sympathy for such drastic measures. The Editors agreed to think it over and to obtain more definite data on these changes. A series of meetings ensued and the result was a decision to recommend to the Board of Directors that the PROCEEDINGS be made over but in its mechanical elements only. The Directors agreed to these changes in May, 1938. In the interim, detailed preparations for these changes have uncovered no evidence which would seem to disagree with the basic plans.

The most important problem at the present time is insufficient space to accommodate all of the material which is accepted for publication. When funds were relatively abundant, a lag of about three to five months from the time a manuscript was received until its appearance in the PROCEEDINGS was considered normal. It compared favorably with other publications of a similar nature using comparable methods of editing and publishing. During our "deficit" years, when

funds for publication were limited, this time lag spread to over a year. During 1936, this condition improved but once more is showing strong evidence of a relapse. The result of this delay in publication is serious not only in that useful engineering data are withheld from the reader but important papers are published elsewhere because more prompt publication can be had. These may be lost entirely to many of our readers and authors are encouraged to think in terms of other journals as outlets for their papers.

There are two main sources of publication funds: the membership and the advertiser. There are four variables: the number of members, the average dues paid by each, the number of advertisers, and the amount spent by each. The difficulties of increasing the membership and the small effect it has on the number of additional pages that can be published, since more copies have to be supplied for an increasing membership, makes that course relatively unresponsive. Similarly, although our dues are about the lowest of any large engineering society, these are inappropriate times to ask for more money. Advertising revenue offers the best source and an examination of the present situation shows certain important factors.

Not being of the popular variety, the PROCEEDINGS will always have a relatively small circulation in comparison with others in the radio field. In general, advertisements are prepared for use in all publications but are designed to fit those magazines having a common page size. The irritation, cost of new art work, and general revision of this copy is, of course, intimately associated with the "special" job for which it must be prepared and is a barrier to the sale of advertising space in the "nonstandard" magazines. All this means reduced income from advertising.

An analysis of 508 magazines carrying advertising and distributed in fields in which engineering is an important factor showed 53 percent using an advertising type page of 7 inches by 10 inches. The other 239 publications were scattered widely over 132 different sizes. The trend towards a standard size is, thus, sharply evident. Discussions with present and potential advertisers confirm the desirability of the more generally adopted size. It is estimated that when the new size is established, the increase in advertising income will be equivalent to a substantial increase in dues and as the money will be spent for more PROCEEDINGS space, the benefit to the membership will be direct.

The outside dimensions of the page are fixed by the mechanical tolerances demanded by commercial printing methods, the smallest size, which is used by any of the publications analyzed, which have the 7-inch by 10-inch type page, is $8\frac{1}{2}$ inches by 11 inches, the standard

letter-sheet size. This fits all standard filing cases and sectional and other bookcases are readily available to accommodate it.

Contributory reasons for this decision are numerous and some are as follows. By far the largest use of the PROCEEDINGS is made at desks or worktables. The new size will stay open at any page without being weighted, something which the smaller size does not do. Folded and pasted inserts are not only expensive but offer problems in binding annual volumes as they are often damaged in trimming the edges. The larger size will avoid many of these. Longer mathematical equations can be handled without being broken. Two columns on each page will make for easier reading and will permit even larger illustrations in most cases than are now used.

In making this change in its publication policies, the Institute has unconsciously followed the history of many of the major engineering societies, none of which have ever reversed the process. Thus we have good reason for anticipating appreciable benefits and conveniences to the readers of the PROCEEDINGS as a result of the impending change.

October and November Meetings of the Board of Directors

The Board of Directors met on October 5 in the office of the Institute. Those present were Haraden Pratt, president; Melville Eastham; treasurer; H. H. Beverage, Ralph Bown, F. W. Cunningham, Alfred N. Goldsmith, O. B. Hanson, Alan Hazeltine, L. C. F. Horle, E. K. Jett, A. F. Murray, B. J. Thompson, H. M. Turner, and H. P. Westman, secretary.

Forty applications for Associate, two for Junior, and four for Student grade of membership were approved.

Approval was granted of a number of manufacturing standards to be submitted to the American Standards Association by the Sectional Committee on Radio.

Permission was granted to the Technical Committee on Electronics to hold an informal conference in New York during January, 1939.

A Technical Committee on Symbols was established to handle both letter and graphical symbols.

Final modifications were made in the proposed amendments to the Institute Constitution which are now to be prepared in printed form for circularization to the membership for balloting.

A memorandum outlining the objectives and program to be followed in the preparation of the annual reviews of 1938 activities was adopted.

An Investments Committee, comprised of Treasurer Eastham as

chairman and Doctors Bown and Goldsmith, was appointed to report to the Board on the present status of the Institute's investments.

The November meeting of the Board of Directors was held on the 2nd in the Institute office. Those present were, Haraden Pratt, president; Melville Eastham, treasurer; E. H. Armstrong, H. H. Beverage, Ralph Bown, Alan Hazeltine, L. C. F. Horle, C. M. Jansky, Jr., A. F. Murray, B. J. Thompson, H. M. Turner, and H. P. Westman, secretary.

It was agreed that, starting in 1939, the Technical Committee on Television and Facsimile would be divided into two committees to treat independently the two subjects now covered by the single committee.

A. L. Green was transferred to Fellow, and J. B. Epperson, A. Lopez, and W. D. Oliphant were transferred to Member grade. H. L. Crowley, G. L. Crossley, S. P. Chakravarti, and R. P. Siskind were admitted to the grade of Member. Twenty-six Associates, four Juniors, and four Students were elected.

As the result of the ballots cast in the election of officers, R. A. Heising was declared elected President for 1939 and P. O. Pedersen, Vice President; V. M. Graham, F. B. Llewellyn, and B. J. Thompson were declared elected Directors to serve for the period 1939-1941.

An invitation to be represented on the Sectional Committee on Electrical Installations on Shipboard—C66 of the American Standards Association was accepted and I. F. Byrnes and F. D. Webster designated to serve thereon.

A committee was appointed to investigate interference in radio reception with the thought of collecting information which would be of use to other groups active in the suppression of such interference.

The Fourteenth Annual Convention will be held in New York City at the Hotel Pennsylvania on September 20–23, 1939. H. P. Westman was designated chairman of the committee in charge of the convention.

In response to a petition, a section of the Institute was provisionally established in Portland, Oregon.

Electronics Conference

An informal conference on the advanced problems of ultra-high-frequency electronics, electron optics, and the electronic problems of high-transconductance devices will be held in New York City about the middle of January, 1939. Final arrangements have not been completed but it is expected that the meeting will occupy two days. Anyone desiring to attend should write the Secretary for further de-

tails and indicate the subject which he will be prepared to discuss. It is expected that the discussions will be of interest chiefly to specialists in these specific fields and no formal papers will be presented.

Committee Work

Admissions Committee

A meeting of the Admissions Committee was held in the Institute office on Wednesday November 2 and attended by H. P. Westman, acting chairman and secretary; Melville Eastham, J. F. Farrington, L. C. F. Horle, C. M. Jansky, Jr., and A. F. Van Dyck. Five applications for transfer to Member grade and two for admission to that grade were approved.

Annual Review Committee

A meeting of the Annual Review Committee was held on October 3 in the Institute office. Those present were A. F. Van Dyck, chairman, E. K. Cohan, J. D. Crawford, assistant secretary; D. E. Foster, J. K. Henney, E. G. Ports, L. E. Whittemore, and H. P. Westman, secretary.

The meeting was devoted to the writing of a memorandum outlining the scope and procedure for the preparation of the annual reviews for 1938.

Board of Editors

On September 28, a meeting of the Board of Editors was held in the Institute office. It was attended by Alfred N. Goldsmith, chairman; R. R. Batcher, P. S. Carter, J. D. Crawford, advertising manager; B. E. Shackelford, and H. P. Westman, secretary.

Further work was done in the preparation for the new format of the Proceedings.

Electronics Conference Committee

On October 27 there was held in the Institute office a meeting of the committee charged with the preparations for the Electronics Conference to be held in New York City in January. F. R. Lack, chairman; R. M. Bowie, J. D. Crawford, assistant secretary; L. F. Curtis, F. B. Llewellyn, G. F. Metcalf, G. A. Morton, B. J. Thompson, and H. P. Westman, secretary, were present.

The scope of the conference and a number of problems pertaining to its operation were discussed. Subcommittees were appointed to consider further some of the problems and prepare recommendations to be acted on by the entire committee.

Investments Committee

The Investments Committee met in the Institute office on November 3 and those present were Melville Eastham, treasurer and chairman; Ralph Bown, and H. P. Westman, secretary. A preliminary examination of the Institute's investments was made and some general recommendations prepared. Data on specific recommendations for submission to the Board of Directors will be gathered prior to the next meeting of the committee.

Tellers Committee

The Tellers Committee comprising L. G. Pacent, chairman; L. A. Kelley, F. M. Ryan, and H. P. Westman, secretary, met in the Institute office on October 28 and counted the ballots cast in the election of officers.

Technical Committee on Radio Receivers

On November 9 the Technical Committee on Radio Receivers met in the Institute office. Those present were D. E. Foster, chairman; C. R. Barhydt, R. I. Cole, L. F. Curtis (representing D. E. Harnett), H. B. Fischer, C. J. Franks, R. S. Holmes, D. D. Israel, J. D. Parker (representing E. K. Cohan), A. E. Thiessen, and H. P. Westman, secretary.

General discussion was given to the preparation of the annual review material on the subjects within the scope of this committee. Consideration was also given to the existing standards and the necessity of expanding the field covered by them.

Technical Committee on Television and Facsimile

On September 29 the Technical Committee on Television and Facsimile met in the Institute office. Those present were H. P. Westman, acting chairman and secretary; R. R. Batcher, G. L. Beers, J. D. Crawford, assistant secretary; K. B. Eller (representing J. W. Milnor), P. C. Goldmark, H. M. Lewis, R. M. Mathes (representing J. L. Callahan), A. F. Murray, and F. W. Reynolds (representing A. G. Jensen). This meeting was devoted to a discussion of the standardization work which comes within the province of the committee. Subcommittees were appointed to prepare preliminary reports.

Institute Meetings

Atlanta

On September 15 the Atlanta Section met at the Acoustic Equipment Company studios, C. F. Daugherty, chairman, presided and there were fourteen present.

"Some Recent Developments in Instantaneous Disk Recording" was the subject of a paper by P. C. Bangs of the Acoustic Equipment Company. Instantaneous and processed recordings were first described and compared. The particular equipment used by this studio was then described including the recorder, audio-frequency amplifiers, the band-pass tuned-radio-frequency receiver used for broadcast program pickup, and the playback and monitoring equipment.

Characteristics of cutting heads, factors governing the number of grooves per inch and the minimum groove circumference, frequency distortion compensation, cutting-head power level, and the weight of the head on the record surface, were all discussed. The recording equipment was then demonstrated on radio programs and recordings of the voices of the various members present at the meeting.

Buffalo-Niagara

The October 15 meeting of the Buffalo-Niagara Section was devoted to an inspection trip to the Buffalo Sewage Disposal plant under the direction of H. C. Tittle, chairman. There were fifteen present.

Chicago

J. E. Brown, chairman, presided at the September 30 meeting of the Chicago Section which was held in Fred Harvey's Union Station Restaurant and attended by 125.

Eugen Mittleman, consultant, presented a paper on "Short-Wave Therapy as a Technical Problem." Dr. Mittelman outlined many problems associated with the use and development of therapeutical short-wave generators. Data were presented to indicate the various amounts of power required to produced desired physiological effects in various parts of the human body. It was pointed out that the patient's sensation to heat is extremely unreliable in controlling dosages and that accurate means of indicating the amount of power absorbed by the patient are essential. A practical method of making direct readings of power absorption was described.

Cincinnati

R. J. Rockwell, chairman, presided over the September and October meetings of the Cincinnati Section which was held in the University of Cincinnati.

At the September 20 meeting, which was attended by 35, Curtiss Hammond of the Ken-Rad Tube and Lamp Corporation presented a paper on "Mixer and Converter Considerations." He divided converter tubes into four general groups. Capacitive coupling and electronic

coupling were discussed and it was pointed out that in certain types of tubes these tend to cancel each other. Mr. Hammond explained also that types having a signal grid in the modulated electron stream tend toward a negative input resistance at high frequencies whereas the reverse is true when the signal grid is next to the cathode. Frequency stability was considered as a function of plate, screen, and anode voltages, and oscillator amplitude.

The October 18 meeting was attended by 70 and S. W. Seeley of the RCA License Laboratory spoke on "Principles and Methods in Television Laboratory Technique." The subject was covered extensively and a few of the major points stressed were the frequency spectrum necessary for picture and voice transmission, the use of a single oscillator for the generation of separate intermediate frequencies for video- and audio-frequency generators, the importance of time when synchronizing, the usefulness of certain types of distortion, and the dangers arising from the prevention of phase shift. The paper was concluded with some demonstrations showing the results of too few lines, too low a frame frequency, interlaced versus sequential scanning, and the formation of ghost images caused by reflections of the signal.

Detroit

On September 23, E. H. I. Lee, chairman, presided at a meeting of the Detroit Section held in the Detroit News Conference Room. There were 60 present.

A paper on "Close-Spacing Antenna Arrays" was presented by J. D. Kraus of Ann Arbor, Mich. It was pointed out that by means of parallel elements in spaces less than one-quarter wavelength apart, substantial gains could be obtained with arrays of relatively small dimension. A number of high-frequency directional systems using closely spaced dephased elements and known as flat-top beams, were described. Curves were presented showing the variation of antenna current and radiation resistance of an array of two dephased elements for different spacing between the elements. Gain-versus-spacing curves were also shown and the effect of antenna loss resistance on gain was illustrated. Curves giving vertical radiation characteristics of a horizontal close-spaced array for various heights above ground as compared to similar characteristics of a horizontal half-wave antenna, were shown. The effect of tilting a horizontal two-element array was also discussed. The talk was concluded with a discussion of practical considerations in the design and construction of efficient close-spaced high-frequency antennas and methods of feeding and adjusting them.

Emporium

On October 6 a meeting of the Emporium Section was held in the American Legion Rooms. A. W. Keen, chairman, presided and there were 48 present.

"Some Notes on the Design and Application of 1.4-Volt Battery Tubes" was presented by E. J. Hoffman of the engineering department of the Hygrade Sylvania Corporation. The need for high efficiency in battery-operated tubes was first discussed. The five types available are: pentagrid converter, 1A7G; diode-triode, 1H5G; radio- or intermediate-frequency amplifier, 1N5G; 100-milliwatt output pentode, 1A5G; and 200-milliwatt output pentode, 1C5G. Design problems of these tubes were explained and they were compared with existing 2-volt tubes. The development of a straight-through filament was of particular interest as it becomes necessary to mount the filament as the final rather than the initial step in construction. All types have a nominal filament rating of 50 milliamperes at 1.4 volts except the 1C5G which requires 100 milliamperes. With the exception of the output tubes, the others operate at zero grid bias. Bias for the output tubes may be obtained from the plate-supply voltage.

Indianapolis

The following three meetings of the Indianapolis Section were held at the Indianapolis Athletic Club.

V. C. McNabb, chairman, presided at the May 20 meeting which was attended by twenty-four.

B. V. K. French of P. R. Mallory and Company, presented a paper on "Radio-Photographic Analogies." It covered many similarities of technique and effects in these two fields. The effects of filters and selective networks were illustrated. The similarity of the mosaic structure of the television image and the screen of a color transparency was shown by colored microphotographs.

The June 1 meeting was presided over by I. M. Slater, vice chairman, and there were 15 present.

D. I. Angus of the Esterline-Angus Instrument Company presented a paper on "Telemetering." At the invitation of Mr. Angus, those present proceeded to his factory in Speedway City where telemetering equipment was demonstrated. The device operates by changing the grid bias on a pair of type-45 tubes. The circuit is arranged to be independent of line resistance and is well adapted to the distant metering of all types of instruments whether they be mechanical, hydrostatic, or electrical. The meeting was concluded with a general tour of the plant in which recording instruments are manufactured.

The September 23 meeting was attended by 28 and presided over by B. V. K. French, secretary-treasurer.

G. E. Landt, technical director of the Continental Diamond Fibre Company spoke on the "Behavior of Solid Dielectrics at High Frequencies." Dr. Landt discussed the various characteristics of insulating materials of interest to the radio engineer. Electrical stability of power factor, dielectric losses, and dielectric constant were discussed as functions of temperature, humidity, and frequency. Comparative tabulations of the characteristics of crystalline and amorphous materials were discussed. Various insulation losses which were discussed include electron drift, atomic vibration, molecular movement, and heterogeneous character theories. Recent refinements of measurement technique to extend the measurements into the ultra-high-frequency region were described. A new amorphous insulation material called "Dilectene" was described which exhibits exceptional stabilized properties at ultra-high frequencies.

Los Angeles

On September 20 a meeting of the Los Angeles Section, which was attended by 175, was held at the KNX transmitter building, in Torrance, Calif. R. O. Brooke, chairman, presided.

A paper on "The New High-Efficiency, 50-Kilowatt KNX Transmitter" was presented by J. L. Middlebrooks, liaison engineer of the Columbia Broadcasting System.

The transmitter plant was opened for inspection prior to the presentation of the paper. Also, motion pictures were shown of the construction of KSFO in San Francisco which was recently completed.

A detailed and comprehensive discussion of the points of interest in the new transmitter was given. The RCA transmitter employs a high-efficiency final amplifier, 3-phase alternating-current filament tubes, and air-cooled 5-kilowatt tubes. The plant is built in an earthquake-proof building and has its own well and water-storage plant, and a gasoline-engine emergency generating system so that operation may be maintained under abnormal conditions. The equipment in the studios in Hollywood may be operated with gasoline-driven emergency power supplies and together with a short-wave link between the studio and transmitter permit the entire station to be independent of all public utilities supplying it.

Montreal

One hundred fifty members and guests attended the October 12 meeting of the Montreal Section at the Engineering Institute of Canada auditorium. Sidney Sillitoe, chairman, presided.

S. T. Fisher, of the special products engineering department of the Northern Electric Company, presented a paper on "The Electrical Production of Musical Tones." The speaker described the various musical scales which have been used since the time of the Greeks and outlined the advantages and disadvantages of each. The present scale is not perfect but is a good compromise for use with instruments such as the organ and the piano which are too difficult to retune every time the performer wishes to change key, a procedure which is necessary for a true harmonious scale.

A Northern-Hammond organ was used to demonstrate the synthesis of conventional and special tones by combining sine waves in varying amounts of fundamental, harmonics, and subharmonics. An oscillograph was used in conjunction with the organ, the better to illustrate the combination of tones. The paper was followed by a short recital to illustrate the possibilities of the instrument.

Philadelphia

On October 6 the Philadelphia Section met at the Engineers Club with H. J. Schrader, chairman, presiding. There were 160 present.

"Recent Advances in Lateral Disk Recording for Direct Playback" was the subject of a paper by H. J. Hasbrouck, Jr., of the RCA Manufacturing Company at Camden. It was pointed out that lateral, or push-pull type of modulation gives low distortion. A frequency range of from 50 to 10,000 cycles is covered with reasonable uniformity by the recording head used. Recording is on metal disks coated with semiplastic material in which a groove is cut by means of a sharp sapphire stylus. The records may be played many times without noticeable impairment of quality.

Analogies were given of mechanical functions of the recording head compared with the electrical functions of a circuit transmitting voice impulses.

A new lateral transcription pickup of light weight and great flexibility, having a permanent diamond point, was described. The response-frequency characteristic was practically flat from 50 to 9500 cycles. The radius of the diamond point is kept to the standard of 0.0023 inch. A compensating network is included to increase the output volume approximately 5 decibels per octave as the frequency is reduced from 800 to 50 cycles. This reproducer illustrates a new departure in mounting a clamped-reed-type armature. Records of a transcript of a musical broadcast were played to illustrate its effectiveness.

E. W. Kellogg gave a demonstration of how words sound when the phonograph record is played in the reverse direction. He had memo-

rized sounds of a number of words played backward and spoke these into a recording microphone. When reproduced in the reverse direction, they were understandable.

San Francisco

The September 21 meeting of the San Francisco Section was held in the Pacific Telephone and Telegraph Company auditorium. Noel Eldred, chairman, presided and there were 102 present.

"Recent Developments in Radio, Television, and Related Fields" were discussed by F. E. Terman, head of the electrical engineering department of Stanford University.

Dr. Terman presented first a detailed account of current trends in broadcast-receiver design. He pointed out that push-button tuning is being included in more of the sets and that automatic frequency control is not necessary when recently designed condensers, the characteristics of which do not change with temperature, are used. The use of permeability tuning was discussed as were features of high-fidelity sets, loud speakers, and vacuum tubes.

In discussing transmitters, he pointed out that high-efficiency amplifier systems are being used in many of the modern installations. Research on new type antennas which are several miles in length and have sharp vertical directional characteristics were discussed and should give great improvement in transoceanic telephony. His paper was concluded with a brief survey of the field of television which covered features of both the RCA and Farnsworth systems.

The October 11 meeting was held jointly with the local sections of several of the major engineering societies and is known as the Annual Joint Engineering Council Meeting. It was presided over by W. C. Smith, chairman of the San Francisco Engineering Council.

A paper on "Sounds, Ears, Noises, and Acoustical Measurements and their Relation to Machinery Quieting" was presented by E. J. Abbott, president of the Physicists Research Company of Ann Arbor, Michigan. Dr. Abbott opened his paper with a description of the characteristics of the ear. It included sensitivity, minimum perceptible amplitude and frequency differences, and loudness judgment. A demonstration of these characteristics was presented. Loudness units and scales and their application to noise measurement were then considered. The paper was concluded with a discussion of numerous problems involved in the reduction of noise.

Noel Eldred presided at the October 18 meeting of the section which was held in the Pacific Telephone and Telegraph Company auditorium and attended by 51.

A paper on "New Tube Developments" was presented by W. R. Jones of the Hygrade Sylvania Corporation. He discussed first the characteristics, applications, and design problems of the new high-mutual-conductance tube, type 1231, designed for use in television amplifiers. He also discussed the five new 1.4-volt low-current-drain tubes for use in battery-operated receivers.

Seattle

On October 7, the Seattle Section met at the University of Washington with A. R. Taylor, chairman presiding. There were 32 present.

At this meeting W. R. Jones of the Hygrade Sylvania Corporation discussed a high-frequency tube of new design. It covered the type 1231 tube which may be used as a triode, tetrode, or pentode. A new physical design limits flexible leads within the tube, thus reducing lead inductance as well as microphonic tendencies. With a mutual conductance of about 6000 micromhos, the tube is capable of considerable gain even when operated into the low-resistance plate loads used in wide-band amplifiers.

Washington

A meeting of the Washington Section was held on October 10 in the Potomac Electric Power Company auditorium. E. H. Rietzke, chairman, presided and there were 150 present.

Don Basim of the engineering department of the Bendix Radio Corporation, presented a paper on "The Theory and Experience of Blind Landing." The general problem was first discussed and various steps taken in the development of a specific system were described. The system is unique in that the plane makes contact with the ground in a flying attitude and at a flying speed. Several hundred completely blind landings have been made with this system in test transport planes. A short motion picture of the apparatus in use during an actual blind landing was projected.

Corrections

W. E. Benham has brought to the attention of the editors the following corrections to his paper, "A Contribution to Tube and Amplifier Theory," which appeared in the September, 1938, issue of the Proceedings on pages 1093–1170.

Page 1118, line three of text should read "Fig. 5" instead of "Fig. 4."
Page 1134, equation (66), sign preceding α^3 term should read $-$ instead of $+$.
Page 1142, line four of footnote, $\bar{\imath}$ should read $\bar{\imath}_1$.

Page 1143, line 2 of footnote, "devised" should read "derived."

Page 1146, equation (91a), sign preceding last term should be + instead of −.

Page 1146, line ten of text "suggest" should read "explain"; line twelve, insert "between $\bar{\imath}$ and i" after 30 degrees; line thirteen, the authors of reference 57 explain that the title of Fig. 6 of their paper should read "Comparison of I and I_0+I_c" rather than "Analysis of the assumed current I"; line thirteen, delete sentence "Similarly, . . . shown"; line sixteen, delete word "however."

Page 1146, footnote should read "vol. 79, no. 477, p. 291" instead of "vol. 79, p. 477".

Page 1149, equation (101), "Fig. 4" should read "Fig. 5."

Page 1159, line eleven, dr/ω_0 should read dr/u_0; following equation (b), $x=2x$ should read $x=2X$.

Page 1166, first paragraph of Appendix II, first line, reference 51 should read 52; second paragraph, units of Λ should be as given on page 1162.

Page 1167, line twelve of text, Υ_{11} should read $\overline{\Upsilon}_{11}$; line fourteen, ( ) should be $\left|\ \right|$.

Appendix III—table facing page 1168, last column, sections 11 and 12, interchange Fig. 4 and Fig. 5.

Errata

Mr. H. S. Loh has brought to the attention to the editors the following corrections to his paper, which appeared on the April, 1938, issue of the PROCEEDINGS on pages 469–474.

The title should read

"On Single and Coupled Circuits Having Constant Response Band Characteristics."

On page 470, line 6, the equation should read

$$F = \frac{2\Delta f}{f_0}.$$

Equation (6) should read

$$y_0 = 2s\sqrt{m}\frac{nQ_1^2}{1-s^2}.$$

Equation (9) should read

$$Q_1 = \frac{1}{\sqrt{3}F}(\cdots).$$

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TECHNICAL PAPERS

A SHORT-WAVE SINGLE-SIDE-BAND RADIOTELEPHONE SYSTEM*

By
A. A. Oswald
(Bell Telephone Laboratories, Inc., New York, N. Y.)

Summary—*There is described briefly a short-wave single-side-band system which has been developed for transoceanic radiotelephone service. The system involves the transmission of a reduced carrier or pilot frequency and is designed to include the testing of twin-channel operation wherein a second channel is obtained by utilizing the other side band.*

The paper indicates the reasons which led to the selection of this particular system and discusses at some length those matters which require agreement between the transmitting and receiving stations when single-side-band transmission is employed.

THE FIRST transoceanic radiotelephone circuit employed the single-side-band method of transmission as a technical necessity to overcome transmission difficulties.[1,2,3,4,5] The short-wave systems which followed within a relatively short time were not confronted with the same transmission problems and all started as double-side-band systems. Advances in the art have made possible the practical application of single-side-band to short-wave systems with the accompanying advantages[6,7] and it now appears that this method of transmission is likely to be adopted rather generally for long-distance radiotelephone circuits.

The Bell System already has three transoceanic short-wave systems operating on a single-side-band basis and the Netherland P.T.T.

* Decimal classification: R410. Original manuscript received by the Institute, April 21, 1938. Presented before Thirteenth Annual Convention, New York, N. Y., June 16, 1938.

[1] H. W. Nichols, "Transoceanic wireless telephony," *Elec. Communications*, vol. 2, pp. 11–31; July, (1923).

[2] H. D. Arnold and Lloyd Espenschied, "Transatlantic radio telephony," *Proc. A.I.E.E.*, vol. 42, pp. 815–826; August, (1923).

[3] R. A. Heising "Production of single side-band for trans-Atlantic radio telephony," Proc. I.R.E., vol. 13, pp. 291–312; June, (1925).

[4] A. A. Oswald and J. C. Schelleng "Power amplifiers in transatlantic radio telephony," Proc. I.R.E., vol. 13, pp. 313–362; June, (1925).

[5] O. B. Blackwell "Transatlantic telephony—the technical problem," *Proc. A.I.E.E.*, vol. 47, pp. 369–373; May, (1928).

[6] R. K. Potter "Transmission characteristics of a short-wave telephone circuit," Proc. I.R.E., vol. 18, pp. 581–648; April, (1930).

[7] A. H. Reeves, "The single-side-band system applied to short-wave telephone links," *Jour. I.E.E.* (London), vol. 73, pp. 245–279; September, (1933).

have circuits in operation between Holland and Java.[8] Others are planning to employ single-side-band transmission for new circuits.

It is the purpose of this paper to describe briefly the system and equipment which has been developed for Bell System services, to state and discuss the considerations which led to the adoption of this arrangement, and to indicate the technical matters requiring agreement between the connecting agencies in respect to several technical requirements which need not be considered when establishing double-side-band circuits.

Since single-side-band transmission offers the possibility of twin-channel operation, wherein a second channel is obtained by utilizing the other side band, this feature is included in the description and discussion although it has not thus far been used in commercial service in this country. Relatively little additional equipment is needed for the second channel but the increased selectivity requirements materially affect the circuit design. The necessary apparatus has been provided for an experimental trial of twin operation under service conditions and these tests are now in progress.

In the interests of simplification and clarity, certain terms, such as "carrier," "conversion frequency," etc., are used throughout this paper in accordance with the definitions contained in Appendix I.

Description

General

In this system a single side band with a reduced carrier is derived, amplified, and transmitted. At the receiver the reduced carrier is separated from the side band by a suitable filter, after which it is amplified independently and then used directly for demodulation purposes or to control local-oscillator frequencies to be used in demodulation. The carrier is also used to control the receiver gain (automatic volume control).

Transmitter System

The arrangement of the single-side-band radiotelephone transmitter used in this system is shown in Fig. 1. The arrangement shown is complete for twin-channel operation. Channel *A* is the upper side band from the first step of modulation. Channel *B* is the lower side band. Three steps of modulation are employed to reach the final frequency, which may be any frequency between 4000 kilocycles and approximately 22,000 kilocycles. The conversion frequency, which in this

[8] N. Koomans, "Single-side-band telephony applied to the radio link between The Netherlands and The Netherlands East Indies," Proc. I.R.E., vol. 26, pp. 182–206: February, (1938).

case is the carrier frequency, for the first modulation step is 125 kilocycles. This frequency is derived from a multivibrator held in subharmonic relation by a 625-kilocycle quartz-crystal oscillator. The first modulators are in duplicate, one for each channel. They are balanced modulators and a high degree of balance is maintained to suppress the conversion frequency in the output. Filters employing

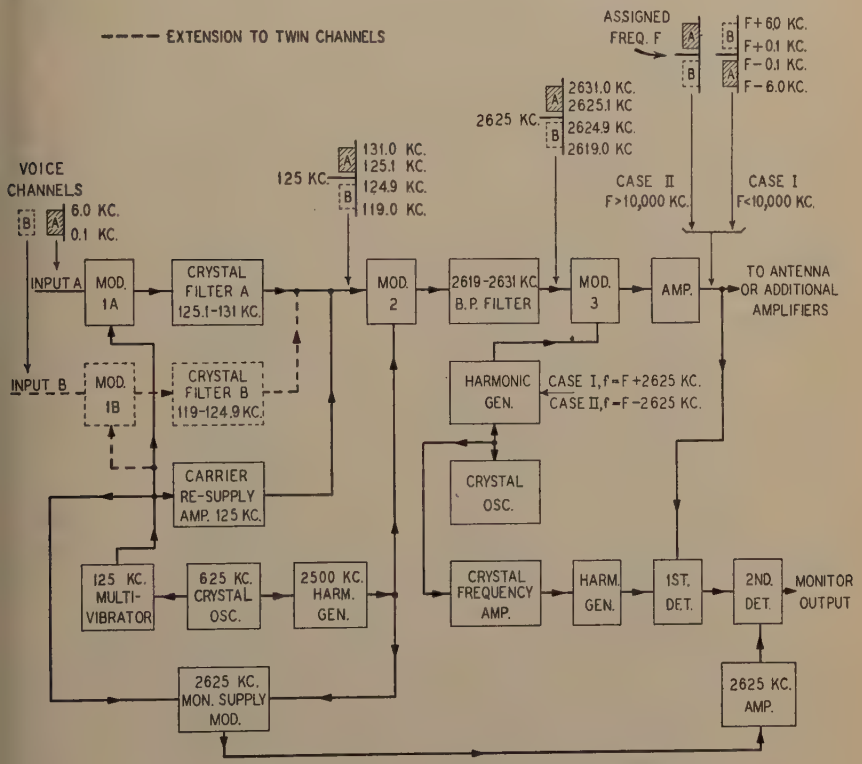

Fig. 1—Block schematic of single-side-band transmitter.

quartz crystals for some of the elements follow each first modulator. The filter in channel *A* selects the upper side band lying between 125.1 and 131 kilocycles. The filter in channel *B* selects the lower side band lying between 119 and 124.9 kilocycles. When only a single channel is wanted the *B* modulator and filter are omitted.

The amplitude of the radiated carrier is controlled by adjustment of the gain of the carrier-resupply amplifier which receives its input from the 125-kilocycle source. The signal outputs of the channel filters and the 125-kilocycle carrier are combined and applied to the second modulator. This is also a balanced modulator and a conversion

frequency of 2500 kilocycles is employed. A harmonic generator driven by the 625-kilocycle oscillator supplies this frequency. The filter following this modulator selects the resultant upper side band, passing the band from 2619 to 2631 kilocycles. The suppression of the lower side band is at least 60 decibels.

The third modulator is not balanced. The frequency of the wanted side-band output from this modulator is sufficiently far removed from the conversion frequency so that the selectivity of ordinary amplifier circuits is sufficient to suppress the conversion frequency and the unwanted side-band products without a special filter. This is an important feature because the circuits between the input of the transmitter and the third modulator remain fixed and are independent of the final radiated frequencies whereas the output circuits of the third modulator and all subsequent circuits require tuning when the radiated frequency is changed. The conversion frequency supplied to the third modulator depends on the final frequency desired. For a radiated carrier frequency f, which is 10,000 kilocycles or above, the conversion frequency is $f-2625$ kilocycles. If f is less than 10,000 kilocycles the conversion frequency is $f+2625$ kilocycles. This conversion frequency is supplied by a harmonic generator driven by a quartz-crystal oscillator. Low-temperature-coefficient crystals are used without temperature control.

Following the third step of modulation a series of amplifiers in cascade raise the level of the signal to the power required. In this transmitting system the final side-band frequency is derived at low power. The vacuum tubes employed in the system up to and including the first final-frequency amplifier are all conventional receiver-type tubes.

A monitor is provided in the transmitter by means of which distortion products in the transmitter output can be measured. A small amount of the transmitter output is combined in a demodulator with the third transmitter conversion frequency so as to produce a side band between 2625 and 2631 kilocycles in the case of channel *A* and between 2619 and 2625 kilocycles in the case of channel *B*. The 2500- and 125-kilocycle conversion frequencies are combined in a separate modulator and the sum is used in a balanced second demodulator to demodulate the 2625-kilocycle side band to reproduce the input frequencies. There are no channel filters provided in the monitor and consequently there is no means of separating the channels should they both be active.

Transmitting Apparatus

As used in the Bell System, the equipment comprises a low-power unit capable of delivering a peak envelope power of 2 kilowatts and a

separate power amplifier for higher-power outputs. The San Francisco-Honolulu circuit employs the low-power unit working directly into the antenna. The New York-London systems use the same unit to drive the power amplifiers formerly used for double-side-band transmission.

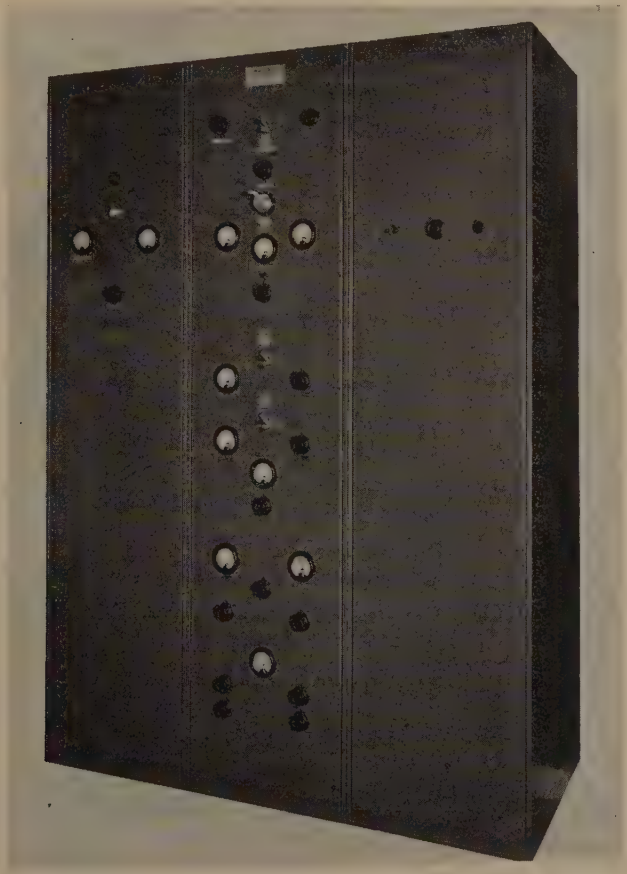

Fig. 2—Front view of transmitter.

Figs. 2 and 3 are photographs of this low-power unit which is similar to that used in earlier experimental work.[9] The apparatus is assembled on 19-inch relay-rack panels mounted in two welded steel cabinets. One cabinet is a single relay rack in width and the other is arranged to accommodate two bays. The smaller unit (left in Fig. 3) houses the

[9] F. A. Polkinghorn and N. F. Schlaack, "A single-side-band short-wave system for transatlantic telephony," Proc. I.R.E., vol. 23, pp. 701–718; July, (1935).

modulators, oscillators, and other low-frequency equipment preceding the third modulator. The latter and the high-frequency amplifier are mounted on the center bay. The third bay (right Fig. 3) contains the rectifiers and other power units. The entire unit operates from 230-volt, 3-phase, 50- or 60-cycle power. The high-frequency equipment

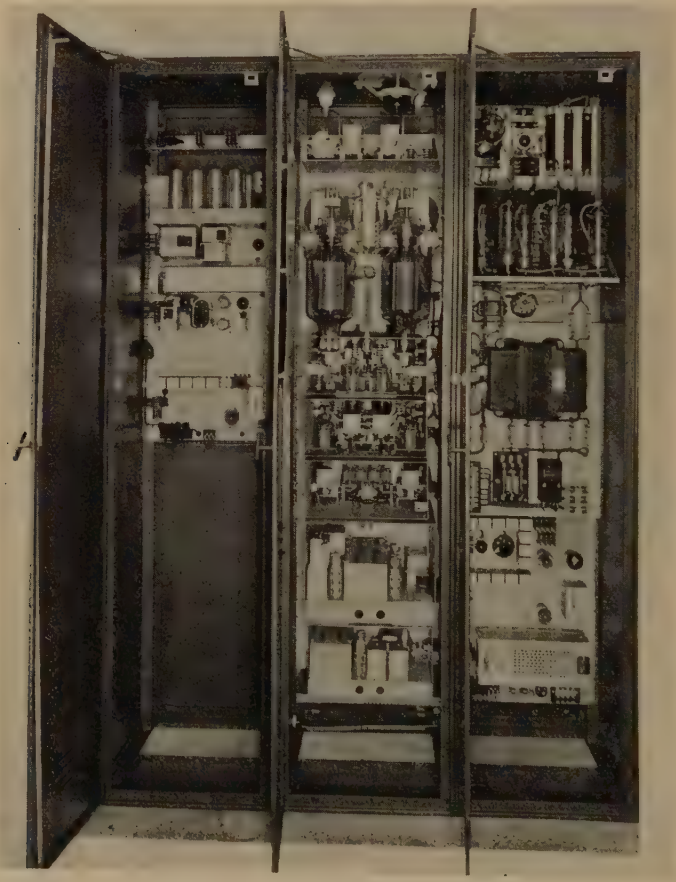

Fig. 3—Back view of transmitter.

is arranged in such a way that interchangeable pretuned circuits and crystals are available by means of selector switches for four frequencies in the lower-powered stages. Continuously variable inductances are used for tuning the higher-power stages. The apparatus may be adjusted to any one of four predetermined frequencies without opening the doors. The pretuned circuits and crystals may be removed and others inserted, without disturbing the adjustments of the units thus readily permitting the use of additional frequencies when desired.

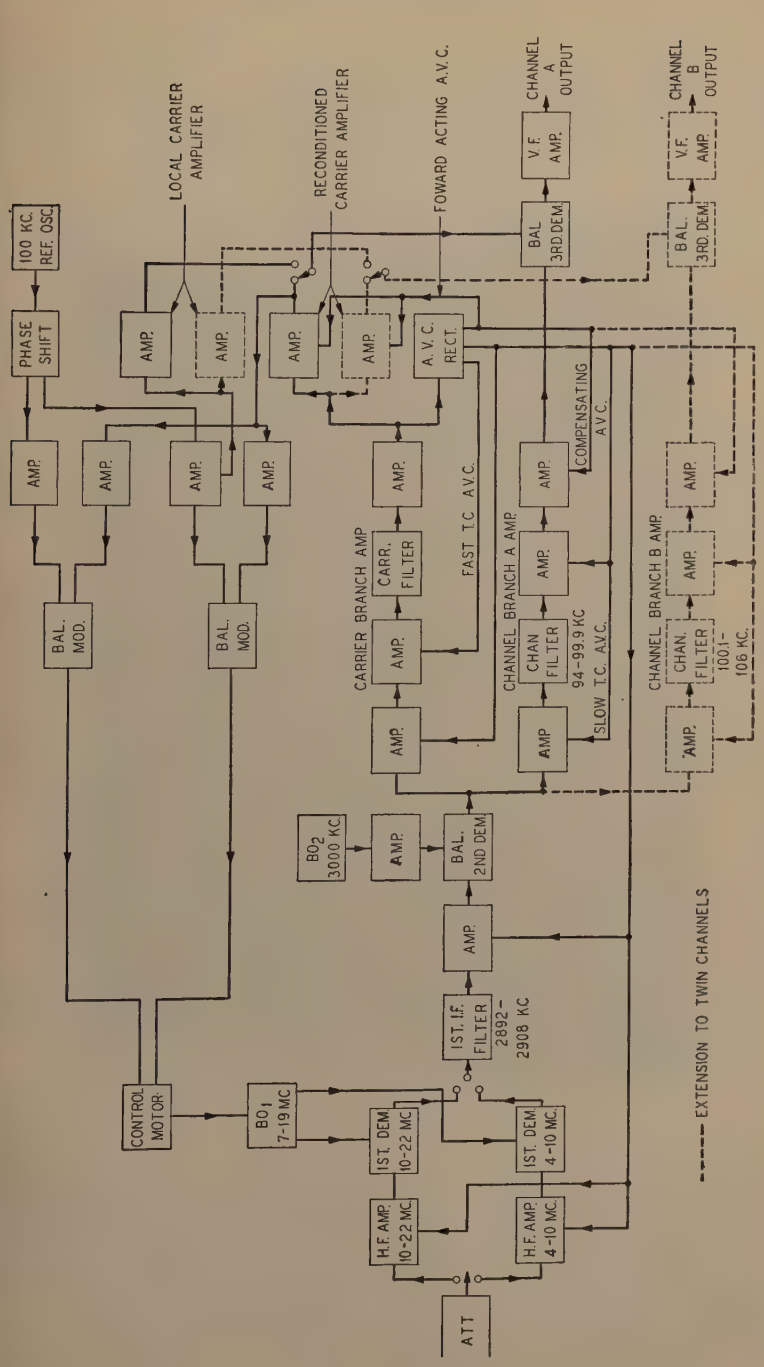

Fig. 4—Block schematic of single-side-band receiver.

Receiving System

The block arrangement of the receiver is shown in Fig. 4. This diagram shows the equipment for twin-channel reception. For single-channel operation the *B*-channel equipment is omitted.

Fig. 5—Front view of receiver.

The receiver is designed to operate at any frequency in the range from 4000 to 22,000 kilocycles. This range is covered by two separate high-frequency amplifiers and first demodulators. In order to attain a high degree of circuit stability, two separate sets of high-frequency equipment are used rather than the more conventional method of changing coils. The received-carrier frequency is translated to 2900

kilocycles in the first demodulators. Following the first demodulator are filters which discriminate against frequencies which would cause interference in the 100-kilocycle second-intermediate-frequency circuits. The relatively low frequency of 100 kilocycles is used for the second intermediate frequency in order to obtain the sharp cutoff characteristics required of carrier and channel filters. A 3000-kilocycle

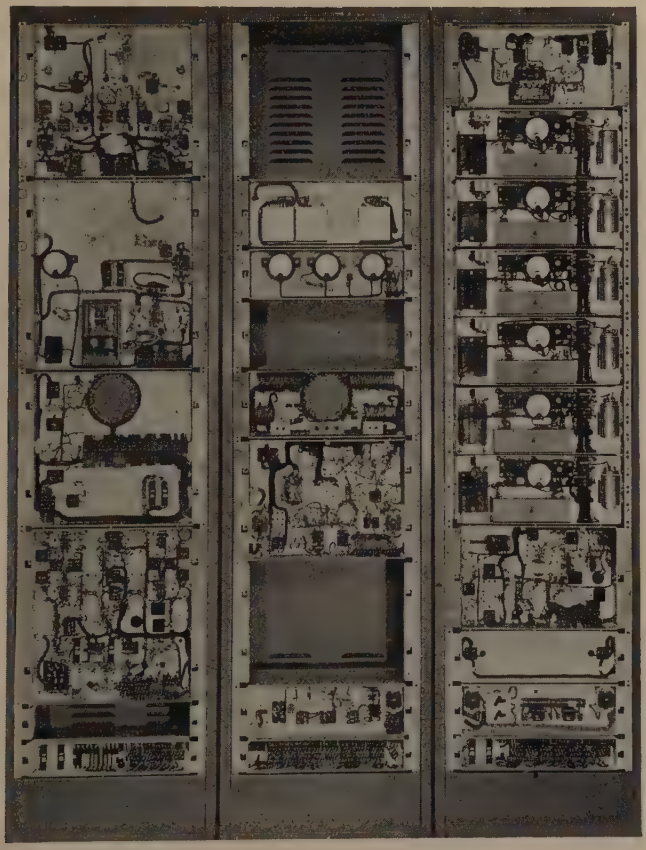

Fig. 6—Front view of receiver with mats removed.

quartz-crystal beating oscillator is used at the second demodulator to translate the 2900-kilocycle carrier to the 100-kilocycle.

Following the second demodulator the circuit is branched to (1) a carrier-reconditioning and automatic-gain-control circuit, (2) the channel-*A* amplifier and filter circuits, and (3) the channel-*B* circuits when required. The carrier-amplifier circuit supplies reconditioned carrier at constant volume for demodulating the signal at the third

demodulators. It also supplies voltages to the gain-control rectifier and to the automatic-frequency-control circuits. Quartz crystals are employed in the carrier filter and for some of the elements of the channel filters. These filters furnish high attenuation to frequencies outside of their respective pass bands, and have extremely sharp cutoff char-

Fig. 7—Back view of receiver with dust covers removed.

acteristics. The band widths are 40 cycles for the carrier filter and 5900 cycles for the channel filters.

In order to maintain the signal in the proper relation to the crystal filters, a system of automatic frequency control of the first beating oscillator is employed. The frequency of the first beating oscillator is adjusted by a control motor in such a manner that the frequency of

the reconditioned carrier is maintained in synchronous relation with that of a high-precision 100-kilocycle quartz reference oscillator. This reference oscillator may also be used to furnish a local carrier for the final demodulation as an alternative to the normal reconditioned carrier.

Receiving Apparatus

The equipment is assembled on 19-inch relay-rack panels arranged in three cabinets. Figs. 5, 6, and 7 are views of this apparatus. Referring to Figs. 5 or 6, and reading from top to bottom, the panels on the left-hand cabinet are (1) the high-frequency amplifiers and first detectors, (2) the first beating oscillator, (3) the first intermediate-frequency filter, amplifier, and second detector, (4) the automatic-tuning-control amplifiers and modulators, (5) a blank filler mat, and (6) a terminal panel for intercabinet cabling.

The panels on the center cabinet are (1) a blank filler mat, (2) the channel filter, (3) meters for the measurement of carrier rectifier current, additional vacuum-tube plate and screen currents, and voice-frequency output volume, (4) space for a second-channel filter, (5) the monitoring panel, (6) the channel amplifier and third detector, (7) space for a second-channel amplifier and third detector, (8) the 100-kilocycle reference oscillator, and (9) a terminal panel. The panels on the right-hand cabinet are (1) the second beating oscillator, (2), (3), (4), and (5) 130-volt regulated rectifiers, (6) and (7) 400-volt regulated rectifiers, (8) the carrier amplifier and automatic-volume-control rectifier, (9) the carrier filter, (10) the main power and fuse panel, and (11) a terminal panel. A screw-driver adjustment and a voltmeter on each rectifier provides a means for setting each rectifier voltage to the proper value. Switches on the power panel permit the receiver to be shut down completely for extended periods or partially shut down during stand-by periods. In the latter case, all power is turned off except that required to maintain the oscillators at operating temperatures.

Discussion

General

There are several methods of generating a single-side-band signal. The following discussion, however, is limited to methods involving simple modulators and filters.

Choice of Intermediate Frequencies in Transmitters and Receivers

The generation of a single-side-band signal by means of simple modulators and filters require that the selection of the side band be accomplished at frequencies where suitable filters can be obtained. If

the radiated frequencies, however, are to be within the range from 4000 to 22,000 kilocycles it is necessary to select the side band, and then translate it by additional steps of modulation to whatever frequency position is desired. If the first step of modulation is accomplished at as high a frequency as possible, it reduces the number of additional modulation steps required to reach a given maximum frequency. It is possible to obtain suitable filters for selecting a single side band at frequencies higher than 125 kilocycles but this appeared to be an economical choice at the time the transmitter was designed. By using first-intermediate-frequency filters near 125 kilocycles, it is possible to translate the side band to frequencies as high as 22,000 kilocycles by utilizing only two additional steps of modulation.

The choice of the second intermediate frequency must represent a compromise. Assuming that it is equally easy to obtain the same percentage selectivity regardless of the absolute frequency, the second intermediate frequency should be the geometric mean between the first intermediate frequency and the highest radiated frequency. On the other hand, it is desirable to facilitate changes in the radiated frequency by using simple circuits where frequency changes have to be made. Simple high-frequency circuits are not very selective. A somewhat higher frequency than the geometric mean is therefore desirable for the second intermediate frequency.

In the case of a receiver for single-channel reception the necessary selectivity usually can be obtained by double demodulation, using an intermediate frequency of the order of 400 kilocycles. For systems requiring more selectivity in order to separate channels lying near each other, such as occurs in the twin single-side-band system, triple demodulation is desirable. When triple demodulation is adopted, the remarks concerning choice of frequencies previously made in connection with the transmitter apply equally to the receiver.

A first intermediate frequency of 125 kilocycles and a second intermediate frequency of 2625 kilocycles were chosen in the design of the single-side-band transmitting equipment. In the receiver, which was designed at a later date, the corresponding frequencies are 100 and 2900 kilocycles. It is not essential, of course, that these same frequencies be used by other connecting radio equipment.

Relation Between Channels and Side Bands

In any single-side-band transmitter where all equipment preceding the last stage of modulation remains fixed regardless of the radiated frequency, the side band and carrier at the input to the final modulator will always appear in the same frequency position. If the conversion frequency for the final step of modulation is placed above the desired

radiated frequency the relative position of side band and carrier will be inverted, but if the conversion frequency is placed below the desired radiated frequency the position will not be changed. In a receiver wherein the second and third demodulators operate at fixed frequencies a similar situation arises if the position of the first-beating-oscillator frequency is changed from a higher frequency to a lower frequency than the received signal.

For radiated frequencies less than twice the intermediate frequency, harmonics of the first beating oscillator are apt to be troublesome if the beating oscillator is below the radiated frequency rather than above it. The same is true in the transmitter. Therefore, it is desirable when working at frequencies less than twice the highest intermediate frequency to use a conversion frequency higher than the radiated frequency.

On the other hand at the higher radiated frequencies it is desirable to use a final conversion frequency less than the radiated frequency. This yields higher frequency stability and a decreased range of oscillator tuning. Interchanging the relative frequency position of the conversion and radiated frequencies at some point in the frequency range reduces the total frequency range which must be covered by the conversion-frequency supply by twice the intermediate frequency. The same remarks apply to the receiver. For example, for a range of radiated frequencies from 4000 to 22,000 kilocycles with a first intermediate frequency of 3000 kilocycles the beating-oscillator range need only be from 7000 to 19,000 kilocycles instead of 7000 to 25,000 kilocycles. As mentioned previously, this change in relative position of the conversion frequency results in an inversion of the radiated carrier and side-band position. This presents no difficulty if the change is made at the same frequency at the transmitter and receiver. This procedure, of course, requires that an agreement be reached between the connecting agencies in regard to the radiated frequency at which the change in the relative position of final carrier and side band is to be made. In the apparatus described, this change can be made anywhere within the range from 10,000 to 15,000 kilocycles but 10,000 is preferred.

In single-channel operation with the present transmitters and receivers, it is planned that for all radiated frequencies less than 10,000 kilocycles the carrier will be placed above the side band. In a twin-channel system the above applies to the *A* channel and the *B* channel will occupy the opposite position.

Frequency Stability

For proper operation of a short-wave single-side-band system, the gain of the receiver must be adjusted to allow for variations in signal

intensity and the frequency of the carrier used for demodulation must be in nearly correct relation to the side band.

Satisfactory intelligibility on a totally suppressed carrier single-side-band radiotelephone circuit without privacy devices will be obtained if the demodulated speech frequencies at the output of the receiving terminal are within about 20 cycles of the original frequencies at the input to the transmitting terminal. When band-splitting privacy systems are used, this tolerance must be reduced to about 5 cycles. If the transmitting-system and receiving-system frequencies are independent, the use of the band-splitting privacy leads to the requirement that the sum of the deviations in frequency from the assigned values of all the oscillators in the transmitting or in the receiving system cannot exceed ± 2.5 cycles.

The requirements upon absolute stability may be relaxed if a reduced carrier or a pilot frequency is transmitted with the side band. At the receiver the carrier is separated from the side band, amplified, and then used either to demodulate the signal or to control the demodulation frequency. This compensates for the deviations of all the oscillators between the point in the transmitting system where the reduced carrier is introduced and the point in the receiving system where the demodulation controlled by the carrier is accomplished. The deviations of any oscillators used for modulation or demodulation before or after these points remain uncompensated.

When a reduced carrier is radiated it may be separated from the side band at the receiver, amplified, and then used in demodulating the side band. Under these conditions the frequency stability of the carrier need be only sufficient to insure that the carrier remains within the band of the filter provided at the receiver to separate it from the side band and from interfering signals and noise. If the frequency of the first or second beating oscillator in the receiver is controlled by the carrier (automatic frequency control), a further reduction in the stability requirements is permissible, it being necessary only that the total frequency deviations be within the capability of the control mechanism. (It should be noted that when some frequency other than the carrier is used as a pilot for control purposes, the frequency-control system is apt to be more complicated.) When automatic frequency control is employed, attention must be given to the rate of change of frequency as well as to the deviation.

To establish a limit for the frequency stability of the receiver it should be observed that the automatic frequency control in the receiver must cover a range equal to the sum of the transmitter and receiver deviations. The maximum rate at which the sum of the transmitter

and receiver frequencies can vary without loss of control is fixed by the constants of the automatic-frequency-control system and by the width of the filter in the carrier branch of the receiver. The present receivers are designed to accommodate transmitter-frequency variations of 1 part in 10^4 with the maximum rate of change less than 5 cycles per second per second. Higher stability is, of course, very desirable and usually is justifiable for other reasons.

Carrier Reduction

As has already been shown, one result of radiating a reduced carrier is to alleviate the frequency-stability requirements. In more complicated receiving systems such as those employing sharp angular directivity and angular diversity,[10] the transmission of the carrier is essential for other important functions quite apart from frequency-stability problems. Another reason for radiating the reduced carrier is that it may be used for automatic-gain-control purposes. In a twin-channel system the carrier is preferred over any other single frequency for automatic gain control because it lies between the two channels.

When the carrier is used for automatic-gain-control purposes at the receiver, the amplitude of the radiated carrier should bear a definite relation to the peak side-band amplitude. Furthermore, the variations in the relation with time should be within reasonable limits, in order that the over-all loss of the circuit for the side band may be maintained as near as possible at the same average value by the automatic gain control of the receiver. It is convenient to express the amplitude of the radiated carrier in reference to the amplitude of the side band produced by one of two equal test tones which, when applied simultaneously, load the transmitter to its rated envelope peak output. (The rated envelope peak output is determined by the distortion requirement.) One reason for choosing the amplitude of either of two equal tones as a reference is that two equal tones are used in making distortion measurements. Another reason is that this reference amplitude is approximately equal to the amplitude of the carrier in double-side-band and carrier transmission when using the same transmitting amplifier and thereby provides a convenient correlation between the two systems. A carrier reduction of at least 10 decibels below reference is required to avoid unnecessary loading of the transmitter. It is desirable to reduce the carrier further in order to minimize unwanted modulation products which will result in cross talk in the twin system.

The maximum amount of carrier reduction is determined by the

[10] H. T. Friis and C. B. Feldman, "A multiple unit steerable antenna for short-wave reception," Proc. I.R.E., vol. 25, pp. 841–917; July, (1937).

ability of the receiver to select the carrier and discriminate against noise and interference. Our experience indicates that the maximum practicable carrier reduction is likely to be about 25 decibels when using a carrier filter having a band width of approximately 40 cycles. For the present in single-channel systems it seems desirable to transmit the carrier at −10 decibels from reference amplitude. In twin-channel systems it is planned to radiate the carrier at −20 decibels. It is desirable, therefore, that means should be provided in the transmitter for varying the carrier amplitude at least between −10 and −25 decibels and there should be corresponding reciprocal arrangements provided in the receiver.

In order to obtain a constant-carrier amplitude in the radio transmitter which is independent of the balance in the first modulators, the first conversion frequency is substantially eliminated and the desired amount of carrier is introduced through an adjustable-gain amplifier. In this manner the carrier amplitude is conveniently adjustable from 0 to −25 decibels. At present it appears reasonable to require that the carrier amplitude at this point be held constant within 1 decibel.

The constancy of carrier amplitude observed at the receiver depends upon three things; first, the constancy of amplitude at the point in the transmitter where the reduced-carrier amplitude is established; second, compression in the amplifier stages through which both the carrier and side band pass; and third, fading due to irregularities of the transmission path. The amount of compression that can be tolerated is not known but certain tone tests have been made which give an indication of the conditions which permit single-channel and twin-channel operation. It has been found that for single-channel service a compression of approximately 3 decibels in the carrier is permissible between the condition of no input and when two tones at reference amplitude are impressed on the transmitter. It is clear that if this condition exists for single-channel operation that there will be momentary compressions of larger value whenever large speech peaks occur simultaneously on the two channels in a twin system. In respect to variations due to the transmission path it is clear that selective fades may cause the received carrier to disappear substantially at times. It is essential from these considerations that the automatic gain control on the carrier amplifier should be separate from that which controls the side-band amplifiers and should operate as fast as possible. Such arrangements are provided in the present receivers.

Carrier-Filter Selectivity

Reception in a single-side-band reduced-carrier system, wherein the reduced carrier is reconditioned or is used to indicate the correct

frequency for demodulation, requires a high degree of selectivity in the carrier filter and imposes definite restrictions on the single frequencies and amplitudes that may be radiated adjacent to the carrier. This latter detail requires special attention at the transmitter in order that the receiver may function properly, and, therefore, requires agreement between correspondents. The carrier filter in the receiver must pass the carrier and discriminate against frequencies in the side band. Any frequencies which fall within the transmission band of the carrier filter must be attenuated to an amplitude which is negligibly small compared to the received-carrier amplitude either by an audio-frequency filter preceding the transmitter or by the filter which selects the single side band in the transmitter, or both.

The band width of the carrier filter must represent a compromise. The wider the transmission band of the filter, the easier it is to keep the carrier in the band. Widening the band increases the vulnerability of the received carrier to noise and interference and tends to limit the amount the carrier may be reduced at the transmitter. Considering these opposing tendencies and judging from our experience with such systems, it appears that a band width of 40 cycles represents a practical compromise.

The present radio receivers are equipped with carrier filters having a mid-band frequency of 100 kilocycles and a band width of 40 cycles. The discrimination to frequencies outside the band is given below:

Frequency Departure from Mid-Band in Cycles	Discrimination
± 25	10 db
±100	60 db

Channel-Filter Selectivity

For single-channel single-side-band operation the discrimination outside the pass-band of the filter, which selects the single side band in the transmitter, must be sufficient to reduce the unwanted side band until substantially no load caused by this side band is imposed on the amplifier stages. From this standpoint a discrimination against the unwanted side band of about 20 decibels or more is satisfactory. More than this is required to reduce interference if the adjacent frequency space is to be used for other purposes. For twin-channel operation, frequencies in the unwanted side band of one channel falling in the band occupied by the other channel should be discriminated against by at least 50 decibels.

One of the filters which selects the desired side band in the radio transmitter, which is designed for twin-channel service, has a pass-

band from 125.1 to 131 kilocycles (Fig. 8). It is uniform within $\pm\frac{1}{2}$ decibel except at the extremes where there is an increased loss of 2 decibels. It presents 20-decibel discrimination at 124.9 kilocycles and at least 50-decibel discrimination from 119 to 124.8 kilocycles. No particular requirements are imposed on this filter above 131 kilocycles. The other filter has a pass band from 119 to 124.9 kilocycles. It is uniform within $\pm\frac{1}{2}$ decibel except at the extremes where there is an increased loss of 2 decibels. It offers 20-decibel discrimination at 125.1 kilocycles and at least 50-decibel discrimination from 125.2 to 131 kilocycles. No particular requirements are imposed below 119 kilocycles.

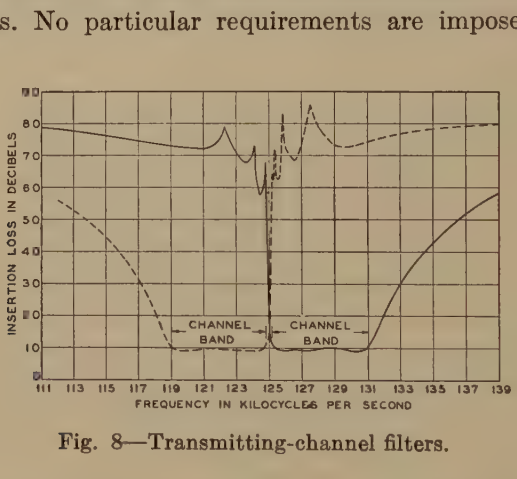

Fig. 8—Transmitting-channel filters.

In the receiver, the channel filters provide the major part of the selectivity. It is not likely that interfering signals adjacent to the desired signal will exceed the desired signal by more than 40 decibels except for a small fraction of time, so that protection against such interference amplitudes will practically eliminate interference from such signals. If interfering signals are discriminated against by the channel filters so that they are 50 decibels below the desired signal, the desired signal will be degraded to a negligible degree by their presence. These final filters should afford, therefore, at least 90-decibel attenuation to all frequencies outside the band which might cause interference.

One of the channel filters in the radio receiver has a pass band from 94 to 99.9 kilocycles (Fig. 9). It is uniform within $\pm\frac{1}{2}$ decibel except at the extremes where there is an increased loss of 2 decibels. It presents 10-decibel discrimination to a frequency of 100.1 kilocycles and at least 90-decibel discrimination to frequencies at or below 93 kilocycles and at or higher than 101 kilocycles. The other filter has a pass band from 100.1 to 106 kilocycles. It is uniform within $\pm\frac{1}{2}$ decibel except at the extremes where there is an increased loss of 2

decibels. It presents 10-decibel discrimination to 99.9 kilocycles and at least 90-decibel discrimination to frequencies at or below 99 kilocycles and at or higher than 107 kilocycles.

Noise

The effect of noise voltages such as power-supply ripple in a radiotransmitter amplifier is to modulate any frequency passing through the amplifier with these voltages. In the case of a conventional double-side-band-and-carrier transmitter the products which cause concern

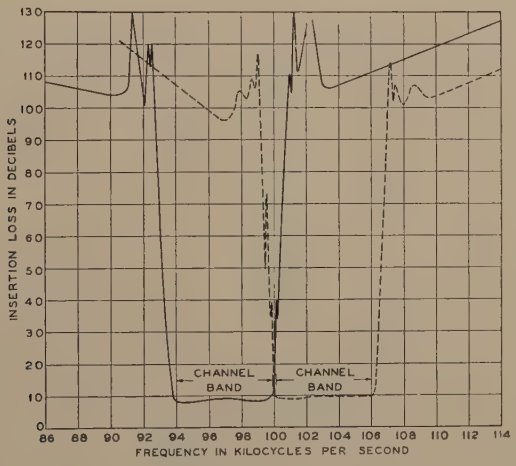

Fig. 9—Receiving-channel filters.

are the resultant double-side-band products of the noise and the carrier.

Likewise in a single-side-band transmitter the noise modulates the reduced carrier and also the side band caused by the signal. In this case, as the carrier is transmitted at a reduced amplitude, both the products with the carrier and the products with the side band must be considered. The products with the carrier are in the form of a double side band on the carrier and are present in the output in the absence of a signal input.

The transmitter signal-to-noise ratio cannot be measured accurately in a monitor in which there is no channel selectivity unless the carrier supplied to the monitor is in phase with the transmitted carrier. However, a satisfactory measurement can be made indirectly by measuring the double-side-band noise modulation with a simple linear rectifier connected to the output of the transmitter. The rectified output con-

sists of a direct current caused by the carrier with an alternating component superimposed. The alternating component in the rectifier output is measured with no input to the transmitter. An input tone is then applied to the transmitter which produces a side-band signal whose amplitude is less than the carrier and less than the maximum side-band amplitude by a known amount. The rectifier output caused by the tone is then measured. The signal-to-noise ratio determined by these measurements is then less than the maximum signal-to-noise ratio by the amount the signal was reduced from the maximum signal output. That is to say, the sum of the signal-to-noise ratio measured in decibels, plus the amount the signal input was reduced, less the compression in the transmitter, is the maximum signal-to-noise ratio when using the carrier level at which the measurement was made. Since this result is based on a double-side-band measurement, the result at the output of the receiving system will be 6 decibels better if the receiving-channel filter completely suppresses one side band.

With our present experience it appears that the maximum signal-to-noise ratio in the transmitter when measured in the above manner should be at least 45 decibels and preferably 50 decibels.

The modulation of a signal which will occur as a result of noise in the transmitter and appears as distortion can be determined in a similar manner. For convenience, the carrier is substituted for the signal and measurements of the noise modulation made at various amplitudes. An input tone less than the carrier and a known amount less than reference amplitude is impressed on the transmitter and the output of the rectifier measured. The ratio of the rectifier tone output to the noise output at each carrier amplitude is determined. The double-side-band noise modulation of the signal at equivalent amplitudes is the sum of this ratio in decibels, plus the amount the tone was reduced from reference amplitude, plus 6 decibels. It is necessary to add 6 decibels because the reference-carrier amplitude is 6 decibels below maximum side-band amplitude. The lower permissible limit of this ratio has not been determined but it has been observed that no serious distortion effects result from this cause if the signal-to-noise ratio measured on a carrier at -10 decibels amplitude is 45 decibels.

In the case of the receiver the noise introduced by the receiver itself theoretically should be limited to thermal noise originating in the input circuit. In the present receivers this theoretical condition is approached within a few decibels for weak-signal inputs. The maximum signal-to-set-noise ratio is limited to approximately 60 decibels in order that signal voltages may be kept below values which would result in cross modulation.

Intermodulation and Twin-Channel Operation

In order that quality and distortion may be observed or measured at the transmitter, as is the usual practice, it is necessary to have a monitor. In such a monitor a portion of the side-band output of the transmitter is combined with the conversion frequencies to demodulate the signal.

The use of a single frequency for testing does not yield significant results in a single-side-band reduced-carrier system. However, two frequencies may be used to secure significant results. Two frequencies of equal amplitude are used for testing, each frequency being half the peak side-band amplitude. When the amplitude of the largest distortion product of these two frequencies is at least 25 decibels below either of them the distortion is low enough so that satisfactory operation of a band-splitting privacy system can be obtained. Twice the maximum root-mean-square power output at which this requirement can be met consistently is called the rated envelope peak output. This requirement must be met also for all smaller amplitudes of the fundamentals.

It has been found that without reducing the amplitude of either twin channel with respect to a single channel, severe cross talk can be avoided by maintaining a separation between the adjacent side-band boundaries equal to the width of one of the side bands, that is, equal to one band width. The third-order products resulting from any two frequencies, p and q, in any one side band, fall in a band extending one band width on each side of that side band from which they originate. Consequently when there is one band width of separation between the side bands these products do not fall into the other side band and produce cross talk. Fifth-order products, which extend for two band widths on each side of the side band in which they originate, will fall in the other side band and must be considered as a source of cross talk. Also the third-order products involving the carrier will result in cross talk. The amplitude of the carrier is, therefore, of great importance in determining the cross talk which will be encountered in twin-channel operation.

The separation between the side bands may be accomplished by one of several methods. A flexible method is the introduction in either or both transmitter input channels of a device to translate the speech band to a higher frequency. A corresponding device is introduced in the associated receiver output channel to restore the speech band to its original position. The band width of the channel filters (100 to 6000 cycles) in the present transmitters and receivers allows a 250- to 3000-cycle telephone band to be shifted to a band from 3250 to 6000 cycles. This shift may be used in either or in both chan-

nels, or intermediate shifts can be used so long as a band-width space between the resulting side bands is maintained.

Advantages of a Wide-Band Twin-Channel System

The principal advantages of this twin-channel system, with wide-band filters, are its simplicity and flexibility. The use of a common carrier simplifies the modulation and demodulation problems and reduces the amount of equipment required. Symmetrical filters having band widths considerably greater than the band to be transmitted permit variations in the side-band arrangement and may afford a means of reducing interference. Insofar as the radio apparatus is concerned, when wide-band filters are provided in the radio equipment, either one of the channels may be used for the transmission of programs.

Appendix I

Definitions

Carrier

The carrier at any point in the system is that frequency which, when combined in a modulator with the signal at that point, will reproduce the original signal frequencies which were applied at the input of the transmitter.

Radiated Carrier

The term radiated carrier is used in this paper to refer to the carrier frequency which is radiated.

Conversion Frequency

The conversion frequency is the frequency supplied to a modulator for the purpose of converting or translating the signal input to a frequency which is a sum or difference of the input frequency and the conversion frequency. The term "beating-oscillator frequency," when referring to the receiver, has a similar meaning.

Reference Amplitude

This is the amplitude of one of two frequencies of equal amplitude which, when transmitted simultaneously, load a single-side-band transmitter to its rated envelope peak output. It is approximately equal to the amplitude of the carrier in double-side-band-and-carrier transmission when using the same transmitting amplifier and therefore provides a convenient correlation between the two systems in addition to establishing the necessary base from which relative amplitudes may be designated.

Reference Carrier

This is a carrier at reference amplitude.

Reconditioned Carrier

Reconditioned carrier is the received carrier which has been separated from the side band, amplified, and otherwise treated so as to make it suitable for subsequent uses in the receiver.

Envelope Peak Output

The envelope peak output of a transmitter is the root-mean-square power during the maximum radio-frequency cycle which occurs in the transmitter. When making two-tone tests this occurs during the coincidence of the peaks of the two test tones.

Compression

Compression is the reduction in the gain of an amplifier due to the presence of a signal of large amplitude.

Appendix II

Approximate Design Requirements of the Present System

Relation Between Channels and Side Bands

It is planned that the lower side band shall be radiated for carrier frequencies less than 10,000 kilocycles and the upper side band shall be radiated at 10,000 kilocycles and above this frequency. In a twin-channel system the above applies to the *A* channel and the *B* channel will occupy the opposite position.

Frequency Stability

The receiver is designed to accommodate a transmitter frequency variation of 1 part in 10^4 with a maximum rate of change of frequency not exceeding 5 cycles per second per second.

Reduced-Carrier Amplitude and Constancy of Amplitude

At present the reduced-carrier amplitude for single-channel systems is -10 decibels from reference carrier. For twin-channel systems it is planned to use a reduced-carrier amplitude of -20 decibels from reference carrier. The average amplitude is maintained within 1 decibel.

Audio-Frequency Band Width

The audio-frequency transmission band of the over-all radio system is from 100 to 6000 cycles. It is uniform within ± 1 decibel except at the extremes where there is an increased loss of 4 decibels. This allows transmission of the following:

(1) One 250- to 3000-cycle speech band.

(2) The same as in (1) but shifted to any frequency position within the filter.

(3) One broad-band channel from 100 to 6000 cycles.

The actual transmission is determined, of course, by the audio-frequency control-terminal conditions.

Apparatus Noise

The signal-to-noise ratio in the transmitter when measured as described should be at least 45 and preferably 50 decibels.

The noise originating in the receiver should be produced mainly in the input circuit except at very high field strengths.

Cross Modulation

Any distortion product produced in the transmitter should be at least 25 decibels down from either of two frequencies, equal in amplitude, which load the transmitter to rated envelope peak output.

At present it appears that for twin-channel operation with adjacent side-band edges 3000 cycles apart, the cross modulation produced by one side band, 3000 cycles wide, in the other side band, 3000 cycles wide, should be 35 to 40 decibels down.

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A SINGLE-SIDE-BAND RECEIVER FOR SHORT-WAVE TELEPHONE SERVICE*

By

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Summary—*A new radiotelephone receiver has been developed for the reception of reduced-carrier single-side-band signals in the frequency range from 4 to 22 megacycles. This receiver employs triple detection in which the first beating oscillator is continuously variable and the second is fixed in frequency. The first oscillator is a very stable tuned-circuit type, the proper adjustment of which is maintained through the use of an improved type of synchronizing automatic-tuning-control system. The second oscillator is crystal controlled. Separation of the carrier and side band is accomplished in the receiver by means of band-pass crystal filters which provide extremely high selectivity. Unusually high stability and selectivity characterize the performance of the receiver.*

SEVERAL years ago the Bell Telephone Laboratories conducted a series of tests involving the use of single side band with reduced carrier on a short-wave radiotelephone circuit. The results of these tests were published[1] in 1935 together with a description of the transmitter and receiver which were used. The tests proved the practicability of single-side-band operation on short-wave circuits and led to the adoption of this type of equipment by the American Telephone and Telegraph Company for use in certain of their overseas radiotelephone plant. Three systems are in service, two with England and the other with Hawaii. A new type of receiver for operation in the range from 4 to 22 megacycles was developed for, and is in operation on, these circuits. This receiver is superior in performance to the one used in the earlier tests, the improvement being due largely to refinements in oscillators, filters, and automatic-tuning mechanism. The receiver is designed so that it may be equipped for single-channel or twin-channel reception. In a twin-channel system, a second channel is obtained by adding another side band on the other side of the carrier.

The physical construction of the receiver involves a number of individual panels mounted in three rack-type cabinets. Figs. 1 and 2 are

* Decimal classification: R361. Original manuscript received by the Institute, July 21, 1938. Presented before Thirteenth Annual Convention, New York, N. Y., June 16, 1938.

[1] F. A. Polkinghorn and N. F. Schlaack, "A single side-band short-wave system for transatlantic telephony," Proc. I.R.E., vol. 23, pp. 701–718; July, (1935).

front and rear views respectively of the receiver, the latter figure showing the insides of the cabinets through opened rear doors. The panels are of the depressed-front type in which the panel-wiring and power-circuit-filtering elements are within the depressed part of the panel and

Fig. 1—Front view of the receiver.

covered by a mat. The various mats provide the finished front surface of the receiver. The cabinets and mats are finished with a metallic gray lacquer and fittings are of dull black and chromium. An interlocking protective system is built into the receiver for the removal of high voltages from any point in the receiver when that point is exposed by the removal of a mat or shield.

General Circuit Description

The separation of side bands and carrier in the receiver requires filters having extremely sharp cutoff characteristics. A satisfactory filter design for this purpose involves the use of quartz-crystal elements

Fig. 2—Back view of the receiver, cabinet doors open and shield covers removed.

for operation in the vicinity of 100 kilocycles. However, it is extremely difficult to develop a sufficient amount of image suppression in the high-frequency circuits of a double-detection receiver if the intermediate frequency is as low as 100 kilocycles. To overcome this difficulty, triple detection is used in this receiver. The carrier at the first intermediate frequency is 2900 kilocycles which is sufficiently high to per-

mit the practical rejection of first intermediate-frequency images by means of the selectivity of conventional high-frequency tuned circuits. At 2900 kilocycles filtering is introduced which discriminates against images of the second intermediate frequency of 100 kilocycles.

Fig. 3 shows the circuit diagram of the receiver in block schematic form. Two sets of high-frequency amplifiers and first detectors are used to cover the frequency range from 4 to 22 megacycles rather than the more conventional method of plug-in type of coils. Low circuit losses and high stability of the high-frequency circuits are achieved in this manner. The first beating oscillator, which is continuously variable in frequency, heterodynes the signal in the first detector to produce the first intermediate-frequency carrier of 2900 kilocycles. After filtering and amplifying, the 2900-kilocycle signal is heterodyned with a 3000-kilocycle beating-oscillator frequency in the second detector to produce the second intermediate-frequency carrier of 100 kilocycles. The 3000-kilocycle second beating oscillator is of the quartz-crystal type and is temperature controlled.

Following the second detector the circuit divides into either two or three branches. The carrier and side band are selected separately in the first two branches. The third branch is an optional feature and provides the means of selecting the second channel if twin-channel operation is desired. The carrier branch circuit selects and amplifies only the 100-kilocycle carrier and provides this frequency as a pilot for automatic tuning control of the first beating oscillator. It also provides reconditioned carrier at high amplitude for final detection of the side band if so desired. The band width of the crystal filter which selects the carrier is very narrow, the cutoff frequencies being 20 cycles to either side of the 100-kilocycle mid-band frequency. Discrimination is 65 decibels or more with respect to 100 kilocycles for all frequencies 100 cycles or more removed from 100 kilocycles. A most important consideration in the design of this filter is the prevention of shifting of the pass band with variations in temperature. The pass band must be fixed at all times with respect to a 100-kilocycle precision type of oscillator which furnishes the reference frequency for automatic tuning control. For this reason the crystals employed in the carrier filter are of a negligible temperature coefficient type.

The channel branch circuit following the second detector selects side-band frequencies lying in the range from 94 to 100 kilocycles. This side band is amplified and then detected by beating in the third detector with either the reconditioned received carrier or a locally generated carrier. A stage of voice-frequency amplification follows the third detector and provides a maximum receiver output of 10 decibels above

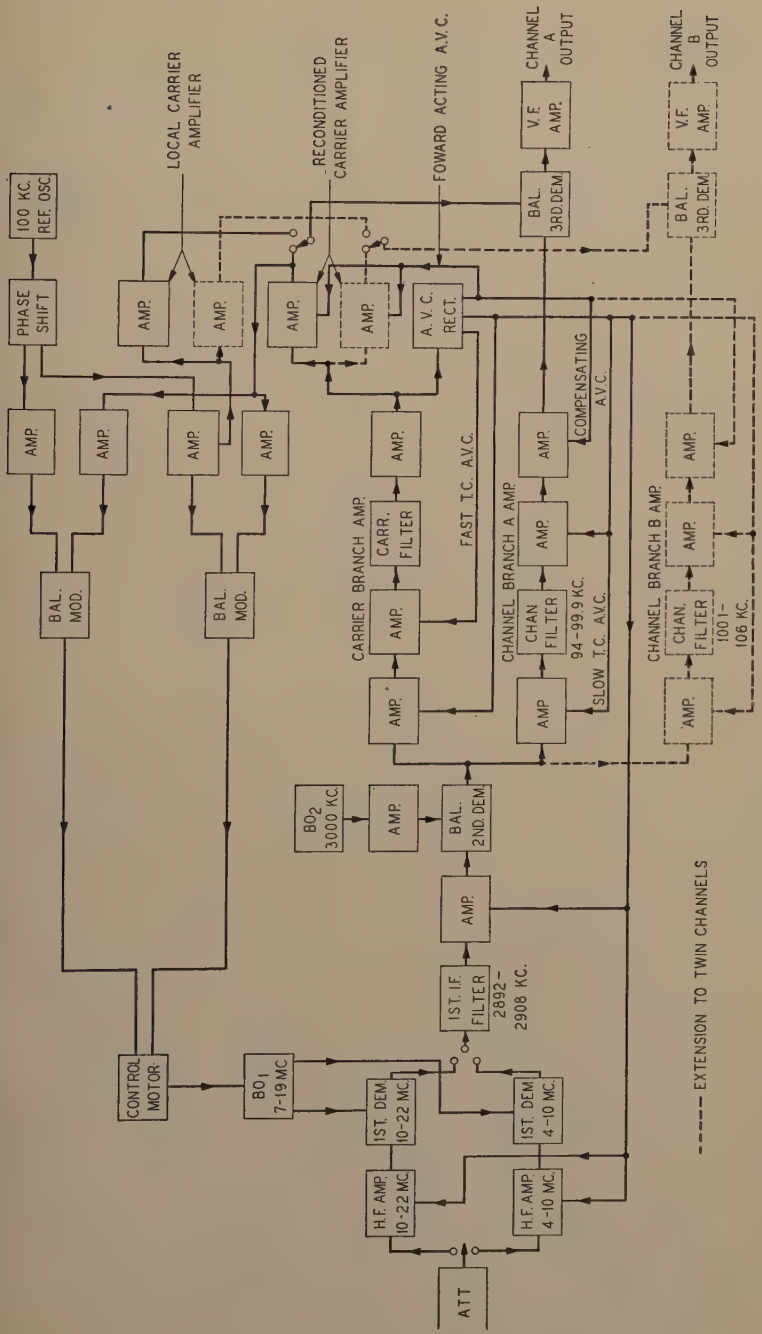

Fig. 3—Block schematic of single-side-band receiver.

6 milliwatts into a 600-ohm load. The filter which selects the side band deserves particular notice, since it is representative of the most recent development of the crystal band-pass filter. It is composed of a four-section lattice network, each section of which employs four crystal elements. The pass band is flat to within plus or minus 1/2 decibel over the entire range of 6 kilocycles. The attenuation rises very sharply at the cutoff frequencies of 94 and 100 kilocycles, the discrimination at 93 and 101 kilocycles being 90 decibels or more. Discrimination of at least 90 decibels is maintained at all frequencies outside of the range of 93 to 101 kilocycles. The channel filter accounts in a large measure for the unusually high degree of selectivity of this receiver.

The optional third branch circuit is a duplicate of the second branch except that the filter pass band is from 100 to 106 kilocycles.

Returning to the carrier branch circuit, the received carrier frequency is rectified to furnish direct voltage for automatic-volume-control purposes. The automatic-volume-control circuit embodies a forward-acting connection which operates in such a manner as to produce an exceedingly flat volume-control characteristic. The design features of this system are discussed at greater length in a subsequent section.

Electrical Design Features

Automatic Tuning Control

In order to permit the use of a locally generated carrier for detecting the side-band signal, the side band must be maintained in an exact relation with the local carrier. This is accomplished by automatically adjusting the first beating oscillator so that the received carrier at the second intermediate frequency is the same as that of the local oscillator. This assures at the same time that the received carrier and side-band frequencies are correct relative to the pass bands of the crystal filters. Electrical reaction types of automatic tuning depend for operation upon the existence of a difference between the controlled and the controlling frequencies and an error is always present when the electrical tuning is functioning. Such control systems are dependent also upon the amplitude of the received signal. Loss of the carrier for an instant, as might occur when a selective fade is encountered, may cause the receiver to become detuned since the oscillator frequency commences to depart from its controlled value as soon as the carrier disappears. The automatic-tuning mechanism used in this receiver overcomes these objections to a large extent. The control system contains a mechanical link which is inoperative when no carrier is present. The mechanical link is a phase-operated motor which actually synchronizes the controlled and the controlling frequencies.

Fig. 4 shows the automatic-tuning-control circuit in block schematic form. A sample of the received carrier from the carrier branch circuit is amplified and then fed to two balanced modulators. Each balanced modulator is also supplied with a voltage from the 100-kilocycle local-carrier oscillator.

In order to produce a two-phase beat-frequency supply for operating a two-phase synchronous tuning motor, the 100-kilocycle oscillator voltages which are supplied to the modulators are displaced 90 degrees

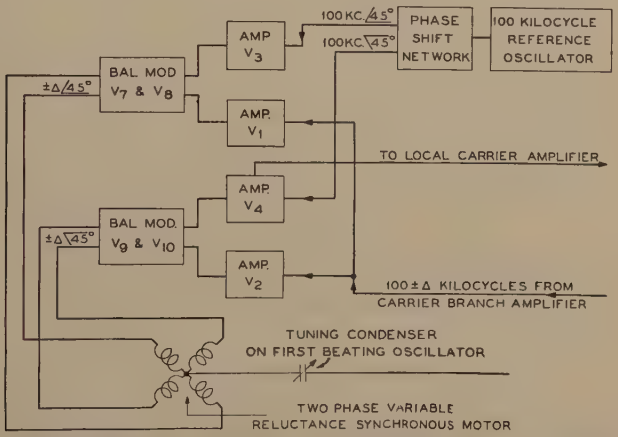

Fig. 4—Block schematic diagram of automatic-tuning-control circuit.

in phase from each other by means of a phase-shifting network. Thus, any difference between the frequencies of the received carrier and the reference oscillator produces a beat frequency in each balanced modulator and these beat frequencies are in quadrature phase relative to each other. This two-phase power at the beat frequency is applied to the tuning motor which in turn makes the small adjustment on the first beating-oscillator frequency to synchronize the received carrier at second intermediate frequency with the 100-kilocycle oscillator. The synchronous tuning motor operates on small fractions of a cycle per second which results in extremely sensitive control. The system maintains the 100-kilocycle received carrier at zero beat with the local carrier oscillator when receiving a signal having normal crystal stability.

First Beating Oscillator

The design of the first beating oscillator is novel in several respects. The fundamental requirement in the design of this oscillator was electrical stability and freedom from mechanical vibration effects. Elec-

trical stability has been obtained by the use of special coils having low temperature coefficients, and by the use of relatively large tuning capacitances. The oscillator circuits are rigidly supported to the inside of a heavy cast-aluminum box. This aluminum casting serves a dual purpose, of electrical shielding and mechanical isolation. The casting is supported from the main panel by means of cushion supports so that it is free to move a limited amount relative to the panel. The mass of the casting and the cushion supports are the elements of a mechanical filter. The automatic-tuning motor is mounted on the oscillator panel immediately below the oscillator casting and the small variable tuning condenser of the motor is coupled to the oscillator circuit through a flexible shielded lead. The beating-oscillator voltage for the first detector is coupled to the modulating grids of the first detectors through a shielded flexible lead. A high degree of shielding and filtering of the oscillators has been maintained in order to prevent radiation and the resultant interference with other receiving equipment in the vicinity. Controls are located on the oscillator panel for manually adjusting the automatic-tuning motor from a 60-cycle supply voltage for setting the initial position of the motor-driven trimming condenser.

Automatic Volume Control

The automatic-volume-control voltage is derived from the incoming reduced carrier which is selected through the carrier branch amplifier. In order that the desired relation between carrier and side-band amplitudes be maintained at the third detector, the volume-control action which applies to the carrier circuits is duplicated as nearly as possible in the side-band circuits. The magnitude of the automatic-volume-control action is the same in the two cases, but the speed of operation is faster for the carrier branch. The design of the volume-control system from receiver input to carrier rectifier is conventional. However, in addition, a limited amount of control is applied to the fourth stage of the carrier amplifier. Since this stage is located at a point in the circuit following the carrier rectifier, it does not contribute to the rectifier input and is therefore forward acting. This type of connection compensates for the inherent rising volume characteristic at the carrier rectifier. A like amount of compensating action is applied to a stage in each channel branch amplifier.

The time constant of the automatic-volume-control system is slow, the value being 8 seconds for all circuits except that of one stage of the carrier branch amplifier. This one stage is supplied with as fast a time constant as the band width of the carrier filter will permit. The 8-second time constant prevents high side-band amplitudes at the third

detector during selective fades of the carrier, and the faster carrier time constant at the same time tends to maintain a constant carrier from the carrier-branch amplifier.

Power Supply

The receiver operates from a 50- to 60-cycle, 110- to 120-volt alternating-current power source. Vacuum-tube cathode heaters obtain power from transformers located on the various panels. Plate current is supplied to the tubes from five automatically regulated rectifiers.

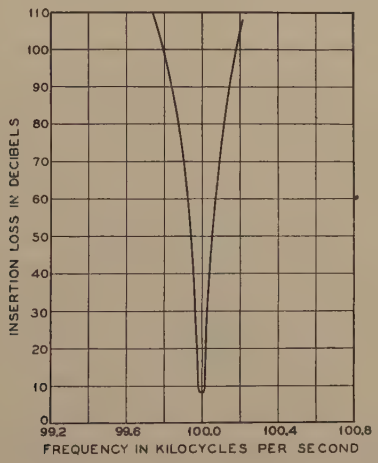

Fig. 5—Pass-band characteristic of carrier filter.

In addition to these a sixth rectifier supplies bias voltages and power for the automatic-volume-control system. The duplication of power rectifiers is for the purpose of minimizing reaction between the various circuit components and contributes appreciably to the high circuit stability obtained.

Metering

All vacuum-tube plate and screen currents may be measured without interrupting the circuit. Each plate and screen current passes through an individual small metering resistor on one of two 36-point metering switches located on the front of the receiver. A meter is associated with each switch, and is transferred from one resistor to another in the manner of a shunting millivoltmeter. By this means it is possible to make routine measurements and to anticipate tube failures, allowing necessary replacements to be made during normal shutdown periods. Other meters provided in the receiver circuit are a carrier-rectifier milliammeter, a voice-frequency output meter and voltmeters on the power rectifiers.

Performance

Figs. 5 and 6 show the attenuation-versus-frequency characteristics of the carrier and channel crystal filters, respectively. In Fig. 6, the optional second-channel filter characteristic is shown in dotted lines.

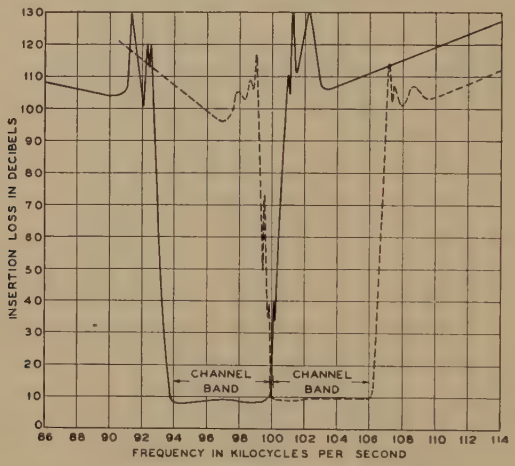

Fig. 6—Channel-filter characteristics.

Sharp cutoff and high attenuation to undesired frequencies are the distinguishing characteristics of these filters, as discussed in a preceding section.

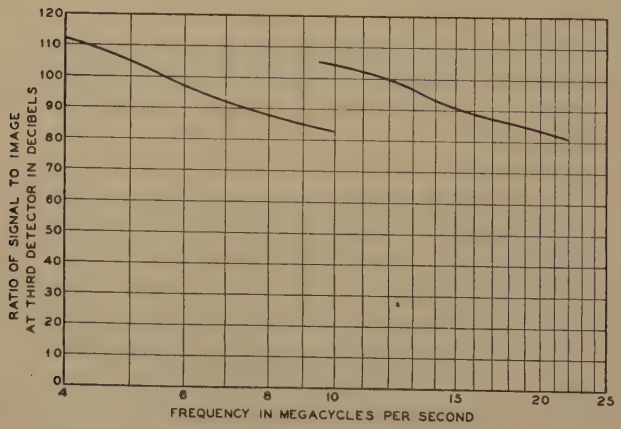

Fig. 7—Image-suppression characteristic.

The noise which originates in the receiver may be classified as thermal, tube, and power noise. The total thermal and tube noise is

approximately 1 decibel greater than theoretical first-circuit noise. Power noise which is principally 120 cycles is down 55 decibels from the signal at the output of the receiver. Thus, in normal operation the signal-to-noise ratio is the ratio of signal to atmospheric noise in the receiving antenna.

Fig. 7 shows the degree of image suppression accomplished by the receiver circuits. The suppression of image frequencies is 80 decibels or more with respect to the desired signal over the entire range of tun-

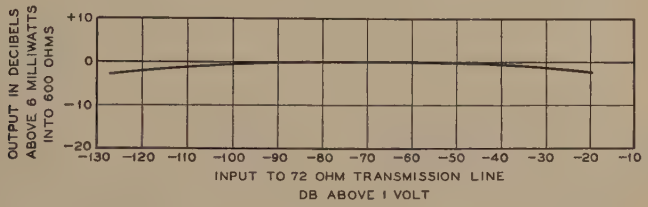
OUTPUT IN DECIBELS ABOVE 6 MILLIWATTS INTO 600 OHMS
+10
0
−10
−20
−130 −120 −110 −100 −90 −80 −70 −60 −50 −40 −30 −20 −10
INPUT TO 72 OHM TRANSMISSION LINE
DB ABOVE 1 VOLT

Fig. 8—Automatic-volume-control characteristic.

ing of the receiver. This suppression is accomplished by means of high-Q radio-frequency circuits, a relatively high first intermediate frequency, and by design of the first intermediate-frequency filter.

Fig. 8 shows the automatic-volume-control characteristic of the receiver. The receiver output is flat to within approximately ± 1 decibel over the useful range of field strengths to be encountered in a commercial circuit. This flatness is obtained by the afore-mentioned forward-acting branch of the volume-control circuit and by initially high gain in the over-all circuit.

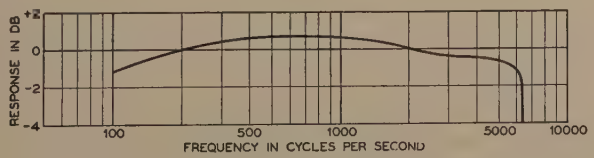

Fig. 9—Over-all voice-frequency—response curve.

Fig. 9 shows the over-all voice-frequency band of the receiver. The nominal band width when receiving a single channel is from 100 to 6000 cycles. The upper limit is determined almost entirely by the channel-filter cutoff.

The initial warm-up period of the receiver is approximately one hour, during which time it is necessary to reset the automatic-tuning motor by manual controls once or twice. After this warm-up period, the receiver stability is such that not more than one adjustment per day is necessary to correct drifting which occurs in the receiver.

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PARALLEL-RESONANCE METHODS FOR PRECISE MEASUREMENTS OF HIGH IMPEDANCES AT RADIO FREQUENCIES AND A COMPARISON WITH THE ORDINARY SERIES-RESONANCE METHODS*

By

D. B. Sinclair

(General Radio Company, Cambridge, Massachusetts)

Summary—*Two parallel-resonance methods are described which determine primarily the parallel conductance of a parallel-tuned circuit. They are most useful for measuring relatively low admittances (high impedances). Since they are duals of the reactance- and resistance-variation methods, they have been named the susceptance- and conductance-variation methods.*

These parallel-resonance methods are compared with the series-resonance methods with respect to range and possible sources of error. It is pointed out that, for substitution measurements, tight coupling to a constant-frequency power source need not introduce errors in measurements with either series- or parallel-resonance methods. The errors caused by residual inductance and metallic and dielectric loss in the standard condenser are discussed and numerical examples given.

A precise method of interpreting resonance-curve data is presented. Experimental results are listed for measurements of high resistances by the susceptance-variation method.

I. Introduction

THE methods in widest use at present for impedance measurements at radio frequencies are probably the so-called "reactance-variation"[1] and "resistance-variation"[2] methods. In their conventional form, these depend upon observation of the current variation in a series-resonant circuit as a function of the change in a known reactance or resistance. They are best suited to the measurement of relatively small impedances.

While such methods may be adapted to the measurement of high impedances, the measurements are usually less accurate than the direct

* Decimal classification: R240. Original manuscript received by the Institute, March 29, 1938. Presented before Rochester Fall Meeting, November 8, 1937.

[1] Originally described by V. Bjerknes, "Ueber electrische Resonanz," *Wied. Ann.*, vol. 55, p. 121, May, (1895). First adapted for use with continuous waves by R. von Traubenberg and B. Monasch, "Über die Verwendung kontinuierlicher elektromagnetischer Schwingungen bei Dämpfungsmessungen," *Phys. Zeit.*, vol. 8, p. 925, December 15, (1907). Also known as the "distuning method" and "capacity-variation method."

[2] Originally described by R. Lindemann, "Über Dämpfungsmessungen mittels ungedämpfter elektrischer Schwingungen," *Verh. der Deutsch. Phys. Ges.*, vol. 11, p. 28, January 15, (1909). Also known as the "added-resistance method."

measurements of low impedances.[3] It is, therefore, highly desirable to have available methods specifically intended for the measurement of high impedances which are as simple and straightforward as the series-resonant methods.

In this paper are described two methods which may be regarded as the "duals"[4] of the reactance- and resistance-variation methods. These methods depend upon observation of the voltage variation across a parallel-resonant circuit as a function of the change in a known susceptance or conductance. They are best suited to the measurement of small admittances. Since they are complementary to the usual series-resonance methods, they have been named the "susceptance-variation" and "conductance-variation" methods.

The series-resonance methods in general use at present are properly methods for determining the effective series resistance of a series-tuned circuit. The parallel-resonance methods to be described are properly methods for determining the effective parallel conductance of a parallel-tuned circuit.

Since all resonance methods primarily determine *circuit* conditions, it should be emphasized that measurements of an *individual* piece of apparatus connected in circuit may be carried out only if the remaining impedances concerned are known. Substitution methods obviate the necessity of detailed knowledge of the elements making up the measuring circuit. That is, if two measurements be made, one with the unknown apparatus in circuit and one with it out of circuit, the admittance or impedance of the unknown may be deduced from the change in circuit admittance or impedance. Since impedances of individual pieces of apparatus are usually desired, the importance of substitution measurements with resonance methods will, accordingly, be stressed.

II. Idealized Theory of Parallel-Resonance Methods

1. The Susceptance-Variation Method

The prototype circuit used for the parallel-resonance methods is illustrated in Fig. 1.

In deriving the simple relationships between conductance, capacitance, voltage, and frequency in the measuring circuit, certain idealized conditions will be assumed initially. The effects of departures

[3] See, for instance, P. B. Taylor, "Method for measurement of high resistance at high frequency," Proc. I.R.E., vol. 20, pp. 1802–1805; November, (1932) and discussion by R. F. Field, p. 1805.

[4] For a discussion of the principles of duality see, for instance, A. Russell, "A Treatise on the Theory of Alternating Currents," vol. I, second edition, chapter XXI, pp. 510–528; Cambridge University Press.

from these idealized conditions will be discussed in a subsequent section.

For a preliminary investigation the circuit residual parameters[5] will be neglected. An imperfect coil will be represented by its equivalent parallel reciprocal inductance Γ_p and conductance G_p.[6] The voltmeter V will be assumed to have negligible admittance and the standard condenser C will be assumed to have a pure capacitance. The power source will be assumed to supply a constant sinusoidal voltage of fixed frequency. Finally, the coupling capacitance C_c will be assumed so small that the current I is essentially constant, irrespective of changes of admittance of the measuring circuit.

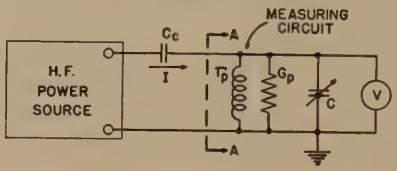

Fig. 1—Circuit for parallel-resonance measurements.

Two readings of voltage and capacitance are necessary with this method to determine the conductance of the parallel-tuned circuit, one at parallel resonance (defined as the point of maximum voltage) and one at any other value of capacitance.

For any arbitrary setting of the standard condenser C,

$$V = \frac{I}{G_p + j\left(\omega C - \frac{\Gamma p}{\omega}\right)}. \tag{1}$$

For the condition of parallel resonance, the susceptance of the measuring circuit is zero, $\omega C = \omega C_r = \Gamma p/\omega$ and the voltage V is a maximum $= V_r$.

$$V_r = \frac{I}{G_p}. \tag{2}$$

Eliminating I between (1) and (2),

$$\frac{V_r}{V} = 1 + j\frac{\omega C - \Gamma_p/\omega}{G_p} \tag{3}$$

[5] Wiring inductance, resistance, and capacitance; mutual inductance and capacitance between circuit elements, etc.

[6] If the coil be assumed to have a series impedance equal to $R_s + j\omega L_s$, as is customary, $G_p = \frac{R_s}{R_s^2 + (\omega L_s)^2} \cong \frac{R_s}{(\omega L_s)^2}$ and $\Gamma_p = \frac{\omega^2 L_s}{R_s^2 + (\omega L_s)^2} \cong \frac{1}{L_s}$.

and expressing (3) in terms of absolute magnitudes rather than complex quantities

$$G_p = \frac{\omega C - \Gamma_p/\omega}{\sqrt{\left(\frac{V_r}{V}\right)^2 - 1}}. \tag{4}$$

But

$$\omega C - \Gamma_p/\omega = \omega C - \omega C_r + (\omega C_r - \Gamma_p/\omega) = \omega(C - C_r).$$

Therefore,

$$G_p = \frac{\omega(C - C_r)}{\sqrt{\left(\frac{V_r}{V}\right)^2 - 1}} \tag{5}$$

where the voltage V_r corresponds to the capacitance C_r and the voltage V to the capacitance C.

An explicit expression, therefore, defines the unknown conductance G_p in terms of a known susceptance difference and a known voltage ratio.[7]

2. The Conductance-Variation Method

If, in addition to the variable condenser, a conductance standard be used, a method of determining the conductance G_p in terms of a conductance difference may be derived. This method requires two readings of voltage at parallel resonance, one with the standard conductance connected in parallel with the measuring circuit and one with it disconnected.

Assume, initially, that the conductance standard has negligible residual parameters and that it may be connected and disconnected from the measuring circuit without appreciably affecting the circuit susceptance.

When the circuit is tuned to resonance (zero susceptance and maximum voltage), (2) shows that the voltage across the measuring circuit is inversely proportional to the circuit conductance. The voltage V_{r_1} across the measuring circuit when the standard conductance is disconnected is, therefore, given by

$$V_{r_1} = \frac{I}{G_p}. \tag{2}$$

[7] This expression has been independently derived by M. Boella, "Un metodo assoluto per la misura della resistenza equivalente dei circuiti oscillatori," *Alta Frequenza*, vol. 4, pp. 647–656; December, (1935); and by L. Hartshorn and W. H. Ward, "The measurement of the permitivity and power factor of dielectrics at frequencies from 10^4 to 10^8 cycles per second," *Proc. Wireless Section I.E.E.*, vol. 12, p. 6, March, (1937). See also W. Anderson, "The dielectric constant and power factor of some solid dielectrics at radio frequencies," *Phil. Mag.*, ser. 7, vol. 13, p. 986, May, (1932).

Similarly, the voltage V_{r_2} across the measuring circuit, when the standard conductance G is connected in parallel, is given by

$$V_{r_2} = \frac{I}{G_p + G}. \tag{2a}$$

Eliminating I between (2) and (2a),

$$G_p = \frac{1}{\frac{V_{r_1}}{V_{r_2}} - 1} G. \tag{6}$$

An explicit expression, therefore, defines the unknown conductance G_p in terms of a known conductance difference and a known voltage ratio.

III. Comparison of Idealized Theories of Parallel- and Series-Resonance Methods

The prototype circuit used for the series-resonance methods is shown in Fig. 2.

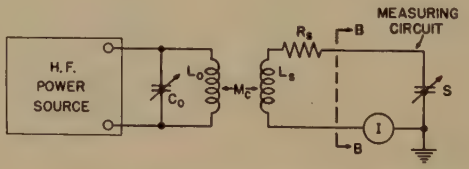

Fig. 2—Circuit for series-resonance measurements.

To illustrate the duality existing between the parallel- and series-resonance methods, the assumptions and mathematical derivations have been collected in comparable form in Table I.

From this table it is seen that strict duality exists between the conductance-variation and resistance-variation methods. For strict duality to exist between the susceptance-variation and reactance-variation methods, the reactance standard used in the reactance-variation method should be a variable inductor rather than the variable condenser. Under such conditions the equation for the reactance-variation method would become

$$R_s = \frac{\omega(L - L_r)}{\sqrt{\left(\frac{I_r}{I}\right)^2 - 1}}$$

which is a complete dual of the susceptance-variation equation.

TABLE I

PARALLEL-RESONANCE METHODS (Refer to Fig. 1) SERIES-RESONANCE METHODS (Refer to Fig. 2)

Assumptions:
1. Circuit residual parameters neglected.
2. Imperfect coil represented by equivalent parallel reciprocal inductance Γ_p and conductance G_p.
3. Voltmeter V has zero admittance.
4. High-frequency power source supplies constant sinusoidal voltage of fixed frequency.
5. Standard condenser C has pure capacitance.
6. Coupling capacitance C_c is so small that current I constant, irrespective of changes of measuring-circuit admittance.

Susceptance-Variation Method

For any setting of C

$$V=\frac{I}{G_p+j\left(\omega C-\frac{\Gamma_p}{\omega}\right)}\cdot \quad (1)$$

At parallel resonance

$$\omega C=\omega C_r=\frac{\Gamma_p}{\omega}$$

and

$$V_r=\frac{I}{G_p}\cdot \quad (2)$$

Combining (1) and (2)

$$G_p=\frac{\omega(C-C_r)}{\sqrt{\left(\frac{V_r}{V}\right)^2-1}}\cdot \quad (5)$$

Conductance-Variation Method

At resonance, standard conductance G disconnected

$$V_{r_1}=\frac{I}{G_p}\cdot \quad (2)$$

At resonance, standard conductance G connected in parallel

$$V_{r_2}=\frac{I}{G_p+G}\cdot \quad (2a)$$

Combining (2) and (2a)

$$G_p=\frac{1}{\frac{V_{r_1}}{V_{r_2}}-1}\,G\cdot \quad (6)$$

Assumptions:
1. Circuit residual parameters neglected.
2. Imperfect coil represented by equivalent series inductance L_s and resistance R_s.
3. Ammeter I has zero impedance.
4. High-frequency power source supplies constant sinusoidal voltage of fixed frequency.
5. Standard condenser S has pure elastance.
6. Coupling mutual inductance M_c is so small that induced voltage E is constant, irrespective of changes of measuring-circuit impedance.

Reactance-Variation Method

For any setting of S

$$I=\frac{E}{R_s+j\left(\omega L_s-\frac{S}{\omega}\right)}\cdot \quad (7)$$

At series resonance

$$\frac{S}{\omega}=\frac{S_r}{\omega}=\omega L_s$$

and

$$I_r=\frac{E}{R_s} \quad (8)$$

Combining (7) and (8)

$$R_s=\frac{\frac{1}{\omega}\,\frac{S_r-S}{\omega}}{\sqrt{\left(\frac{I_r}{I}\right)^2-1}}\cdot \quad (9)$$

Resistance-Variation Method

At resonance, standard resistance R disconnected

$$I_{r_1}=\frac{E}{R_s}\cdot \quad (8)$$

At resonance, standard resistance R connected in series

$$I_{r_2}=\frac{E}{R+R_s}\cdot \quad (8a)$$

Combining (8) and (8a)

$$R_s=\frac{1}{\frac{I_{r_1}}{I_{r_2}}-1}R\cdot \quad (10)$$

IV. Deviations in Actual Circuits from Idealized Theory of Resonance Methods

Expressions have now been derived for the circuit parallel conductance in terms of susceptance or conductance differences and voltage ratios, and for the circuit series resistance in terms of reactance or resistance differences and current ratios. In the idealized cases considered, the circuit conductance or resistance has been assumed to be that of the coil alone.

In addition, simple expressions for the parallel reciprocal inductance or series inductance of the coil have been derived in terms of a standard capacitance or elastance. Measurements under such conditions, therefore, define the electrical properties of the coil, as an individual piece of apparatus, as well as those of the circuit as a whole. In practice, however, the idealized conditions do not obtain and there will, in general, be deviations from the simple theory, caused by circuit residual parameters and by finite coupling to the source, which have not so far been considered.

1. Errors in Parallel-Resonance Methods Caused by Finite Coupling

The errors arising from finite coupling will first be considered. If the coupling is not very weak, variations of admittance or impedance

of the measuring circuits will be "reflected" back into the output of the source. If the source be a self-excited oscillator, these variations will generally cause changes in both frequency and amplitude of oscillation.

In particular, with any of the resonance methods, changes of frequency may lead to serious errors.[8] It is, therefore, highly desirable to interpose between the oscillator and measuring circuit an isolating or "buffer" tuned amplifier. If, for instance, a well-shielded, class A, tuned-radio-frequency pentode amplifier be used, the variation of frequency with load may usually be made negligible, although appreciable variations in amplitude may occur when the measuring circuit is adjusted. Errors caused by frequency changes will then be negligible and

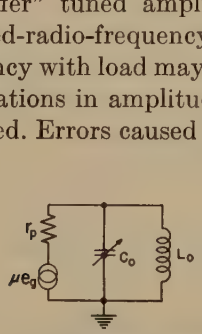

Fig. 3—Equivalent circuit of a high-frequency power source.

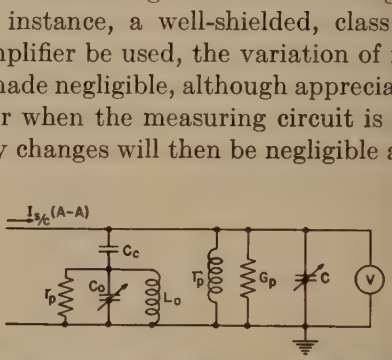

Fig. 4—Equivalent circuit of a parallel-resonance setup, including the effect of finite coupling to a high-frequency power source.

it will be shown that no error is caused by amplitude changes provided the resonance methods are used for determining the admittance or impedance of an *individual* piece of apparatus.

A source composed of an oscillator and isolating amplifier may, to a first approximation at least, be represented by the familiar equivalent circuit of Fig. 3. Consider, now, the case of the parallel-resonance methods, with the circuit of Fig. 1. The assumption made in setting up the circuit relations for the parallel-resonance methods was that the current flowing into the measuring circuit was constant. The equivalent circuit illustrated above is one having a constant-voltage generator. It may, however, be replaced by another equivalent circuit having a constant-current generator which will satisfy the assumption regarding constant current into the measuring circuit. The transformation is carried out by applying a dual form of Thévenin's theorem. This dual form states that any active, linear, two-terminal network is equivalent, as seen from the terminals, to a constant-current, zero-admittance generator, in shunt with a constant admittance. The constant current is equal to the short-circuit current at the terminals with all internal

[8] For a discussion of errors caused by variations in frequency see, for instance, E. B. Moullin, "Notes on the detuning method of measuring the high-frequency resistance of a circuit," *Exp. Wireless and Wireless Eng.*, vol. 7, p. 367, July, (1930); also, "The Theory and Practice of Radio-Frequency Measurements," second edition, Charles Griffin and Company, Ltd., (1931), pp. 279–281.

electromotive forces zero. The circuit of Fig. 1 may, therefore, be redrawn as in Fig. 4 to include the effect of finite coupling.

The circuit of Fig. 4 satisfies the original assumption regarding constant current into the measuring circuit. The circuit conductance, however, is now not the coil conductance G_p alone, but the sum of this conductance and that of the circuit composed of the coupling capacitance C_c in series with the source output impedance. Similarly, the total susceptance of the equivalent circuit is now not zero when the capacitive susceptance of the standard condenser is equal to the inductive susceptance of the coil but when it is equal to the algebraic sum of the coil susceptance and the susceptance of the circuit composed of the coupling capacitance in series with the source output impedance.

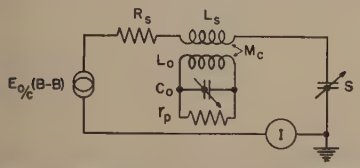

Fig. 5—Equivalent circuit of a series-resonance setup, including the effect of finite coupling to a high-frequency power source.

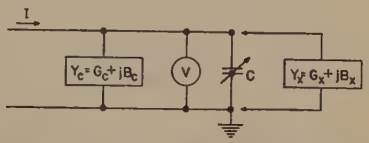

Fig. 6—Simplified equivalent circuit of a parallel-resonance setup.

The direct application of parallel-resonance methods for measuring coils, as outlined for the idealized case, therefore, generally involves errors in the deduction of both the coil conductance and reciprocal inductance. These errors depend upon the output impedance of the source and upon the coupling capacitance C_c.

2. Errors in Series-Resonance Methods Caused by Finite Coupling

A similar analysis may be applied to the case of the series-resonance methods, with the circuit of Fig. 2. For these methods the assumption made in setting up the circuit relations was that a constant electromotive force was induced in series with the measuring circuit. The source may again be represented by the equivalent circuit of Fig. 3. A direct application of Thévenin's theorem immediately leads to the equivalent circuit of Fig. 5, which satisfies the original assumption regarding constant-series electromotive force. The circuit resistance, however, is now not the coil resistance R_s but the sum of this resistance and that coupled into the measuring circuit from the source output impedance. Similarly, the total reactance of the equivalent circuit is now not zero when the capacitive reactance of the standard condenser is equal to the inductive reactance of the coil, but when it is equal to the algebraic sum of the coil reactance and that coupled into the measuring circuit from the source output impedance.

The direct application of series-resonance methods for measuring coils, as outlined for the idealized case, therefore, generally involves errors in the deduction of both the coil resistance and inductance. These errors depend upon the output impedance of the source and upon the coupling mutual inductance M_c.

3. Significance of Errors Caused by Finite Coupling

Before proceeding to a discussion of errors introduced by residual impedances, it seems proper to point out that in measuring *individual* pieces of apparatus the errors caused by change in amplitude have no significance. In the parallel-resonance methods, for instance, the circuit conductance is measured first with the unknown apparatus connected in parallel, then with it disconnected. The effect of finite coupling is simply to introduce an effective admittance, in shunt with the measuring circuit, which depends only upon the output impedance of the source and upon the coupling. Provided the coupling is not varied between the two measurements, the difference between the two measured conductances is, therefore, equal to the conductance of the unknown apparatus, regardless of the magnitude of the conductance component caused by strong coupling to the source. Similarly, the change in resonant capacitance of the standard condenser is an accurate measure of the susceptance of the unknown apparatus since the condenser is set to reduce the total circuit susceptance to zero, both when the unknown is connected and disconnected.

For measurements of the admittance of individual pieces of apparatus, it is, therefore, permissible to consider the circuit of Fig. 1 as simply a two-terminal linear active network, in parallel with the standard condenser C. The equivalent circuit is illustrated in Fig. 6. Since, in this form, the equivalent circuit conforms to the original assumptions in the idealized case, the effective conductance and susceptance of the two-terminal network are given by the expressions

$$G_c = \frac{\omega(C - C_r)}{\sqrt{\left(\frac{V_r}{V}\right)^2 - 1}} \tag{5}$$

$$B_c = -\omega C_r \tag{11}$$

for the susceptance-variation method, and by the expressions

$$G_c = \frac{1}{\frac{V_{r_1}}{V_{r_2}} - 1} G \tag{6}$$

$$B_x = -\omega C_r \tag{11}$$

for the conductance-variation method.

If G_c' and B_c' be the circuit conductance and susceptance with the unknown disconnected and G_c'' and B_c'' the circuit conductance and susceptance with the unknown connected, the unknown conductance G_x and susceptance B_x are given simply by

$$G_x = G_c'' - G_c' \tag{12}$$

$$B_c = B_c'' - B_c'. \tag{13}$$

Similar reasoning applied to the case of the series-resonance methods indicates that no error caused by finite coupling need occur in measuring impedances of individual pieces of apparatus. The circuit resistance is measured first with the unknown apparatus connected in series, then with it disconnected, or short-circuited out. The effect of finite coupling is simply to couple into the measuring circuit an impedance which depends only upon the output impedance of the source

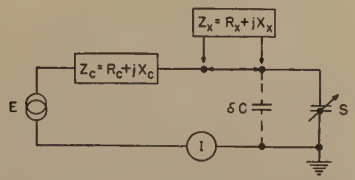

Fig. 7—Simplified equivalent circuit of a series-resonance setup.

and upon the coupling. Provided the coupling is not varied between the two measurements, the difference in the two measured resistances is, therefore, equal to the resistance of the unknown apparatus, regardless of the magnitude of the resistance component caused by strong coupling to the source. Similarly, the change in resonant elastance of the standard condenser is an accurate measure of the reactance of the unknown apparatus since the condenser is set to reduce the total circuit reactance to zero both when the unknown is connected and disconnected.

For measurements of the impedance of individual pieces of apparatus, it is therefore permissible to consider the circuit of Fig. 2 as simply a two-terminal, linear, active network, in series with the standard condenser S. The equivalent circuit is illustrated in Fig. 7. Since, in this form, the equivalent circuit conforms to the original assumptions in the idealized case, the resistance and reactance of the two-terminal network are given by the expressions

$$R_c = \frac{(S_r - S)/\omega}{\sqrt{\left(\frac{I_r}{I}\right)^2 - 1}} \tag{9}$$

$$X_c = -S_r/\omega \tag{14}$$

for the reactance-variation method, and by the expressions

$$R_c = \frac{1}{\frac{I_{r_1}}{I_{r_2}} - 1} \tag{10}$$

$$X_c = -\frac{S_r}{\omega} \tag{14}$$

for the resistance-variation method.

If R_c' and X_c' be the circuit resistance and reactance with the unknown disconnected and R_c'' and X_c'' the circuit resistance and reactance with the unknown connected, the unknown resistance R_x and reactance X_x are given simply by

$$R_x = R_c'' - R_c' \tag{15}$$

$$X_x = X_c'' - X_c'. \tag{16}$$

4. Errors in Parallel-Resonance Methods Caused by Residual Parameters

In the idealized case, for the parallel-resonance methods, the distributed capacitance of the coil, the wiring capacitance, and the admittance of the vacuum-tube voltmeter were all neglected. These admittances are all in parallel with the standard condenser and consequently may be lumped in the two-terminal network of Fig. 6 and included as part of Y_c.

When making measurements of an individual circuit element by the substitution method outlined, these residual admittances do not cause error since they drop out in taking circuit conductance and susceptance differences.

This reasoning applies equally well to the effect of lead resistances and self-inductances except those between the unknown impedance and the standard condenser. Any resistance or inductance in these leads will, of course, be measured as a part of the unknown.

In the case of mutual inductances, errors may arise if electromotive forces are induced magnetically in the loop comprising the standard condenser and the voltmeter. These errors may generally be made negligibly small by the use of short leads and proper shielding.

The most serious source of error lies in the residual impedances inherent in the standard condenser. As was pointed out in a previous paper,[9] a variable air condenser may be represented to a first approxi-

[9] R. F. Field and D. B. Sinclair, "A method for determining the residual inductance and resistance of a variable air condenser at radio frequencies," Proc. I.R.E., vol. 24, pp. 255–274; February, (1936).

mation by the equivalent circuit of Fig. 8(a). Except for second-order corrections, this is the same as the circuit of Fig. 8(b).

In these figures, R represents the effective resistance corresponding to joulean losses in the metallic structure, L, the effective inductance corresponding to the magnetic flux set up by currents in the metallic structure, and G, the effective conductance corresponding to hysteretic losses in the dielectric supports. These may all be considered roughly constant, independent of the dial setting, at any one frequency. C represents the static capacitance of the condenser.

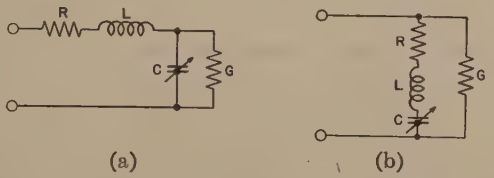

Fig. 8—Equivalent circuits representing a standard condenser with residual inductance and metallic and dielectric losses.

Consider, now, the errors caused by the residual parameters R, L, and G depicted in Fig. 8(b). The dielectric conductance G is directly across the measuring circuit and may be lumped into the admittance Y_c, along with the voltmeter and coil admittances, the admittance component coupled into the measuring circuit from the generator, and the measuring-circuit residual admittances. It, therefore, drops out in the measurement of individual circuit elements by a substitution method. The metallic resistance R introduces a conductance component in the effective circuit conductance which varies with setting. The conductance component arising from this resistance is approximately equal to $R(\omega C)^2$. Except in the very special case when the susceptance of the unknown is zero, this component will cause error in the determination of the unknown conductance since the settings of the standard condenser with the unknown in and out of circuit will be different. The residual inductance L causes the effective capacitance at the condenser terminals to differ from the static capacitance by an amount which depends upon the setting. The effective capacitance is approximately equal to $C/(1-\omega^2 LC)$. Except in the trivial case when the unknown susceptance is zero, the residual inductance will therefore cause error in the determination of the unknown susceptance. In the susceptance-variation method it will also cause error in the determination of conductance, since it will cause error in capacitance differences.

A concrete example will illustrate the type and order of magnitude of the errors discussed. A typical General Radio type 222-M precision

condenser has a residual inductance of 0.0604 microhenry and a metallic resistance of 0.017 ohm at a frequency of 1.5 megacycles. Suppose such a condenser be used as standard to measure a 500-micromicrofarad condenser with a power factor of 0.05 per cent at 1.5 megacycles. Then $C_x = 500$ micromicrofarads, $G_x = 2.36$ micromhos. With the unknown out of circuit, let the setting of the standard condenser for resonance be 1000 micromicrofarads. The effective capacitance at this setting will be 1005.4 micromicrofarads and the conductance component caused by R will be 1.51 micromhos. With the unknown connected, the setting of the standard condenser for resonance will be 504.1 micromicrofarads, corresponding to an effective capacitance of 505.4 micromicrofarads, and the conductance component caused by R will be 0.38 micromhos. The difference between the two resonance settings, as read from the dial, is 495.9 micromicrofarads, which differs from the true effective capacitance increment by 0.8 per cent. The change in the circuit conductance when the unknown condenser is connected is not equal to the conductance of the unknown, but the algebraic sum of this conductance and the change in conductance of the standard condenser. This algebraic sum is 1.23 micromhos instead of 2.36 micromhos. The assumption of zero or constant conductive component in the standard condenser will therefore lead to an error of 48 per cent in the measurement of conductance or power factor of the unknown condenser.[10]

5. Errors in Series-Resonance Methods Caused by Residual Parameters

Turn now to a consideration of the series-resonance methods. An analysis of the effects of various residual parameters again emphasizes the duality with the parallel-resonance methods.

In the idealized case, the wiring resistance and inductance and the ammeter impedance were all neglected. These impedances are all in series with the standard condenser and consequently may be lumped in the two-terminal network of Fig. 7 and included as a part of Z_c.

When making measurements of an individual circuit element by the substitution method outlined, these residual impedances do not cause error since they drop out in taking circuit resistance and reactance differences.

This reasoning applies equally well to the effect of the coil distrib-

[10] This large error in power-factor measurement is the same type of error often found in measurements of dielectrics at high frequencies. In many cases it is found that measurements of power factor tend to be abnormally low, or even negative, at very high frequencies. See, for instance, J. G. Chaffee, "The determination of dielectric properties at very high frequencies," Proc. I.R.E., vol. 22, pp. 1009–1020; August, (1934).

uted capacitance (between terminals). Capacitance from the "high" side of the coil to ground will cause error if it becomes appreciable compared to the capacitance of the standard condenser and if the impedance of the ammeter is not negligible. The error, however, may generally be made negligibly small by shielding the coil L and connecting the shield to the "low" side of the coil. The capacitance of the shield to ground will simply shunt the ammeter and cause no error. Capacitive coupling between the coils in the high-frequency power source and the measuring circuit will cause error but such an error may be eliminated by placing a grounded shield between the coils. Any capacitance between the terminals to which the unknown is connected and between the lead wires to the unknown will, of course, be measured as a part of the unknown.

There is one other capacitance in the series-resonance circuit which will cause error, namely, the capacitance to ground from the side of the unknown which is connected to the standard condenser. This capacitance, denoted as δC in Fig. 7, causes an error in the effective elastance of the standard condenser of a similar nature to the error in effective capacitance caused by residual inductance in the parallel-resonance methods. The effective elastance of the standard condenser is equal to $S/(1+(\delta C)\ S)$ and is seen to differ from the static elastance by an amount which depends upon the condenser setting. Except in the trivial case when the unknown reactance is zero, the residual capacitance will, therefore, cause error in the determination of the unknown reactance. In the reactance-variation method it will also cause error in the determination of resistance since it will cause error in elastance differences.

Consider, now, the errors caused by the residual parameters R, L, and G, illustrated in Fig. 8(a). The metallic resistance R and residual inductance L are directly in series with the measuring circuit and may be lumped into the two-terminal impedance Z_c, along with the ammeter and coil impedances, the impedance component coupled into the measuring circuit from the generator, and the measuring-circuit residual impedances. They, therefore, drop out in the measurement of individual circuit elements by a substitution method. The dielectric conductance G introduces a resistive component in the effective circuit resistance which varies with the setting. The resistive component arising from this conductance is approximately equal to $G(S/\omega)^2$. Except in the very special case when the reactance of the unknown is zero, this component will cause error in the determination of the unknown resistance, since the settings of the standard condenser with the unknown in and out of circuit will be different.

As before, a concrete example will serve to illustrate the type and order of magnitude of the errors discussed. Suppose the precision condenser previously described be used to measure a 2500-micromicrofarad condenser with a power factor of 0.05 per cent at a frequency of 1.5 megacycles. Then $S_x=400$ (microfarads)$^{-1}$ and $R_x=0.0212$ ohm. This precision condenser has a dielectric conductance G of 0.21 micromho at a frequency of 1.5 megacycles, and the setup is so arranged that the residual capacitance δC is 2.4 micromicrofarads. Let the setting of the standard condenser for resonance with the unknown out be 10,000 (microfarads)$^{-1}$ ($C=100$ micromicrofarads). The effective elastance at this setting will be 9766 (microfarads)$^{-1}$, and the resistive component caused by G will be 0.225 ohm. The setting of the standard condenser for resonance with the unknown connected will be 9581 (microfarads)$^{-1}$, corresponding to an effective elastance of 9366 (microfarads)$^{-1}$ and the resistive component caused by G will be 0.207 ohm. The difference between the two resonance settings as read from the dial is 419 (microfarads)$^{-1}$, which differs from the true elastive increment by 4.8 per cent. The change in the circuit resistance is not equal to the resistance of the unknown, but the algebraic sum of this resistance and the change in resistance of the standard condenser. This algebraic sum is 0.0032 ohm instead of 0.0212 ohm. The assumption of zero or constant resistive component in the standard condenser will therefore lead to an error of 84.9 per cent in the measurement of resistance or power factor of the unknown condenser.

V. Comparison of Series- and Parallel-Resonance Methods

The conductance-variation method described, while theoretically useful, is not recommended from a practical standpoint since reliable standards of high resistance appear to be difficult to realize physically.[11] The remainder of this paper will, accordingly, deal chiefly with the susceptance-variation method and its dual, the reactance-variation method.

1. Range and Precision

The range and precision of resonance methods are closely related. Consider, first, the susceptance-variation method. Assume that a pure conductance is to be measured. The lower limit of measurable conductance is determined by the ability to distinguish between the breadths of two resonance curves which are very nearly equal. The

[11] See, for instance, M. Boella, "Sul comportamento alle alta frequenze di alcuni tipi di resistenze elevate usate nei radio-circuiti," *Alta Frequenza*, vol. 3, p. 132, April, (1934); and O. S. Puckle, "The behavior of high resistances at high frequencies," *The Wireless Eng.*, vol. 12, pp. 303, June, (1935).

higher limit is determined by the ability to obtain a large enough part of the resonance curve to interpret. In other words, the lower limit is determined by precision of reading and the upper limit by condenser range.

Now turn to the reactance-variation method. The lower limit of measurable resistance is determined by the ability to distinguish between the breadths of two resonance curves which are very nearly equal. The higher limit is determined by the ability to obtain a large enough part of the resonance curve to interpret. In other words, as before, the lower limit is determined by precision of reading and the upper limit by condenser range.

The susceptance-variation method is best suited to the measurement of low conductances and the reactance-variation method to the measurement of low resistances. There will ordinarily be an overlapping region where either method will give satisfactory results.

2. Accuracy

The accuracy of measurement with the susceptance-variation method depends primarily upon the accuracy with which voltage ratios and susceptance differences are known.

Voltage ratios may be conveniently read on a vacuum-tube voltmeter. Since only ratios need be known, the indication of such a voltmeter need not be independent of frequency, provided the *shape* of the calibration curve does not change.

Susceptance differences are determined from frequency and from effective capacitance differences. Available methods of measuring frequency accurately and of keeping frequency constant assure negligible error in frequency determination. Effective capacitance differences, however, depart from static capacitance differences because of residual inductance in the standard condenser used. In addition, the effective conductance of the measuring circuit changes with condenser setting because of loss in the metallic structure of the standard condenser. The errors caused by residual inductance and metallic resistance increase with frequency and, for any given standard condenser, fix a top frequency above which it is not feasible to operate.

The accuracy of measurement with the reactance-variation method depends primarily upon the accuracy with which current ratios and reactance differences are known.

Current ratios may be conveniently read on a thermocouple milliammeter. Since only current ratios need be known, the indication of such a milliammeter need not be independent of frequency provided the *shape* of the calibration curve does not change.

Reactance differences are determined from frequency and from effective elastance differences. As mentioned above, frequency need not be considered as a factor in determining accuracy. Effective elasance differences, however, depart from static elastance differences because of residual capacitance (δC in Fig. 7). In addition, the effective resistance of the measuring circuit changes with condenser setting because of loss in the dielectric supports of the standard condenser. The error caused by dielectric conductance decreases with frequency. It would, therefore, appear that the reactance-variation method is better adapted to measurements at very high frequencies than the susceptance-variation method.

This conclusion, however, is subject to serious qualifications. Measurements are usually made with standards of smaller capacitance at high frequencies than at low frequencies since the breadth of a resonance curve corresponding to a given conductance or resistance decreases inversely with frequency and this factor tends to aid the parallel-resonance methods in comparison with the series-resonance methods.

If it be assumed, for instance, that ωC be held constant as the frequency is increased and that the residual inductance and metallic resistance decrease proportionally with C, then the inductive error in the susceptance-variation method does not depend upon frequency and the resistance error decreases with frequency.

In contrast, if it be assumed that S/ω be held constant as the frequency is increased and that the residual capacitance and dielectric conductance decrease inversely as S increases, then both the capacitive and the conductive errors in the reactance-variation method are independent of frequency.

The assumptions made above are not, of course, strictly true but it is clear that the frequency limitations of the two types of methods need not differ appreciably if the measuring circuits are properly designed.

VI. Experimental Investigation

1. General Procedure

The experimental work to be described was largely confined to determining the possible range and accuracy which could be obtained with the susceptance-variation method when as many disturbing factors as possible were minimized.

The procedure adopted was to measure several wire-wound high-resistance units in order to determine the range of satisfactory operation and then to compare results obtained with different variations of

the prototype circuit of Fig. 1 in order to determine the consistency which might be expected. A check on accuracy was obtained by measuring the resistance of a straight-wire resistance standard, the skin effect of which could be computed for the frequency used.

2. Interpretation of Experimental Observations

Before the experimental results are discussed a word about the method of interpreting data is in order. Theoretically, two sets of measurements of voltage and capacitance, one at resonance and one off resonance, are all that are required in order to make a conductance determination. Practically, however, much greater precision may be obtained by running over the resonance curve and taking several sets of readings.

A simple method of taking data with the susceptance-variation method, for instance, is to read the voltage at resonance and the two capacitance settings at which the voltage is reduced by $\sqrt{2}$. If C_1 and C_2 be the two capacitance settings for which this condition obtains, (5) becomes

$$G_e = \frac{\omega(C_1 - C_2)}{2}.$$

This method of interpretation is fairly precise because the capacitance values are read under such conditions that small errors of setting are readily seen in the voltage reading.

More precise interpretation may be secured, however, by taking a number of points on the resonance curve and plotting the resulting information in the form of a straight line. From (5) it is seen that a plot of the quantity $\sqrt{(V_r/V)^2-1}$ as a function of capacitance will give such a straight line. The inverse slope of the straight line is equal to the quantity G_e/ω and the intercept with the capacitance axis is equal to the resonant capacitance C_r.[12] Typical plots are shown in Figs. 10 to 14.

While the labor involved in a single measurement is somewhat greater with such straight-line plots than with readings taken at $V = V_r/\sqrt{2}$, the interpretation of data is more satisfactory for precise results. Not only are several experimental observations weighted in the final result, but any deviation from a normal resonance curve may be detected immediately from curvature in the locus of the plotted

[12] E. B. Moullin (footnote 8) describes such a method of interpretation for the reactance-variation method in his "Theory and Practice of Radio Frequency Measurements," second edition, p. 279. His "rectified resonance curve," however, is a straight line only for sharp resonance curves. If his "$\tan\theta(=\sqrt{(I_r/I)^2-1})$ be plotted as a function of elastance, rather than capacitance, a true straight line will result, the inverse slope of which equals ωR_e.

points. In particular, the accuracy depends directly upon the measurement of the voltage at resonance. If this voltage be incorrectly determined, the plot will become distinctly S-shaped instead of linear.

3. Experimental Setup

The setup used consisted of a high-frequency source having an output impedance of about 10 ohms and a high degree of frequency stability, a vacuum-tube voltmeter giving an indication substantially independent of supply voltage variations, and an accurately calibrated precision variable air condenser.

The physical arrangement was such that the unknown could be plugged into circuit in any one of the positions shown in Fig. 9.

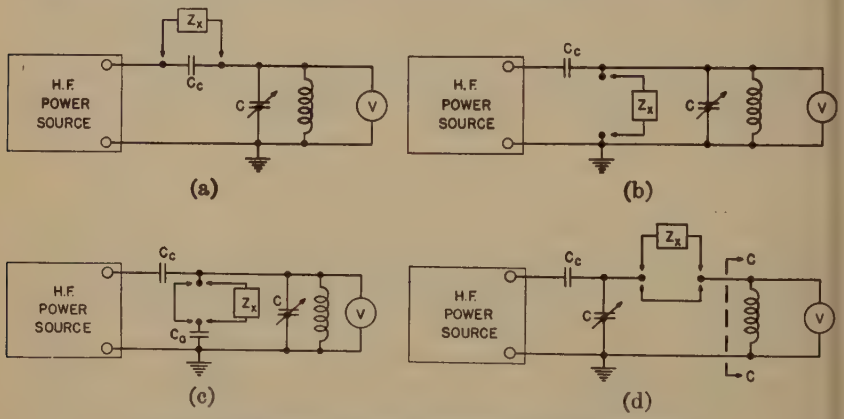

Fig. 9—Variations of the prototype parallel-resonance circuit used for making measurements.

The residual parameters of the variable-air condenser were determined by the method outlined in a previous publication.[9] The data which follow are corrected for errors caused by these residual parameters.

4. Consistency Tests

(a). Measurement of a 10,000-Ohm Resistor

The effective conductance G_e of the measuring circuit was first determined with a small air condenser used for the coupling capacitor C_c. The straight-line plot of $\sqrt{(V_r/V)^2-1}$ as a function of capacitance is shown in Fig. 10.

The unknown was next plugged into circuit in parallel with C_c, as shown in Fig. 9(a), and the effective conductance again determined.

The circuit conductance, in this case, is augmented by the effective conductance of C_c and Z_x, in parallel, in series with the oscillator output impedance.[13] Since the source impedance is only 0.1 per cent of the unknown, the unknown may be considered for all practical purposes as effectively shunting the measuring circuit directly. The straight-line plot of $\sqrt{(V_r/V)^2-1}$ as a function of a capacitance is shown dotted in Fig. 11.

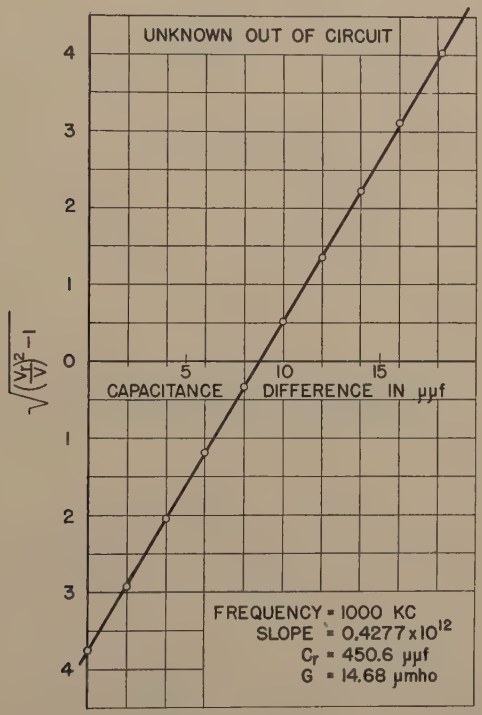

Fig. 10—Plot of resonance-curve data with a 10,000-ohm resistor out of circuit.

The unknown resistor was then plugged into circuit directly across the standard condenser, as illustrated in Fig. 9(b) and the effective conductance redetermined. It was necessary to increase the capacitance C_c in this case, in order to maintain a suitable deflection on the vacuum-tube voltmeter. The capacitance, however, was still sufficiently small so that the change in the conductance component coupled into the measuring circuit from the source output impedance was negligibly small. The straight-line plot of $\sqrt{(V_r/V)^2-1}$ as a function of capacitance is shown solid in Fig. 11.

[13] See Fig. 4 and discussion in Section IV, part 3, regarding error caused by finite coupling.

The difference in conductance with the unknown in and out was 89.5 micromhos for the circuit of Fig. 9(a) and 89.7 micromhos for the circuit of Fig. 9(b). The first of these figures is low by 0.1 per cent because of the output impedance of the source and the final results are seen to agree to about 0.1 per cent.

While the effective conductance component contributed by the unknown should theoretically be the same when connected as shown

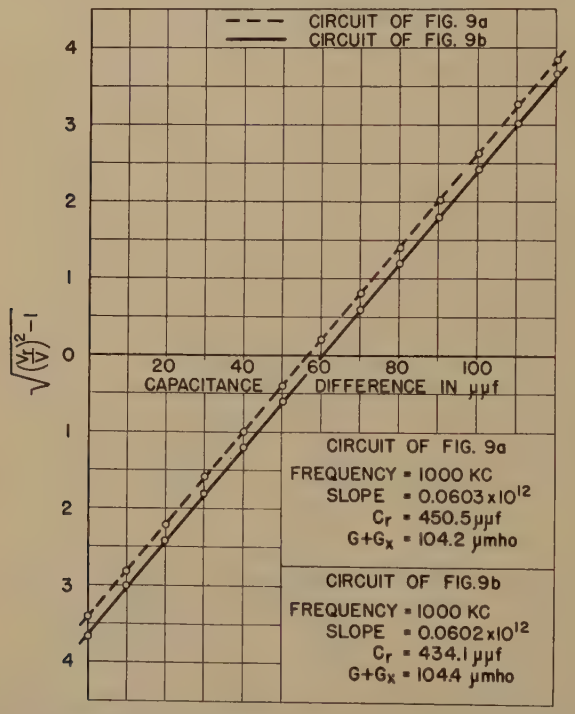

Fig. 11—Plot of resonance-curve data with a 10,000-ohm resistor connected.

in Figs. 9(a) and 9(b), the actual circuit conditions are quite different in the two cases and the experimental verification would seem to indicate that no factor of importance had been omitted from the analysis.

The average value of 89.65 micromhos determined for the card corresponds to a resistance of 11,160 ohms. This value indicates an increase in resistance at a frequency of 1 megacycle of about 1.8 per cent over the value of 10,960 ohms measured at direct current. The measured susceptive component was of the order of 0.5 micromho, capacitive.

(b). Measurement of a 1000-Ohm Resistor

A 1000-ohm resistance card of the same type was measured next. The procedure followed was exactly the same as that outlined for the measurement of the 10,000-ohm resistor.

In this case, the conductance measured with the circuit of Fig. 9(a) was distinctly lower than that measured with the circuit of Fig. 9(b) since the correction for the 10-ohm source output impedance was about ten times greater than it was in the 10,000-ohm measurement. When the correction was applied, however, the values of the unknown resistance deduced from the two measurements were brought into agreement within 0.2 per cent at a value of 1009 ohms. This value indicates an increase in resistance at a frequency of 1 megacycle of about 0.9 per cent over the value of 999.7 ohms measured at direct current. The measured susceptive component corresponds to an effective series inductance of 30.3 microhenrys.

(c). Measurement of a 300-Ohm Resistor

A 300-ohm resistance card of the same type was measured next. With the particular setup used, this value was about the lowest that could be measured directly.

The measurement was first made with the circuit of Fig. 9(b). Since the impedance of the measuring circuit was heavily shunted down by the unknown resistor, it was found necessary to use a high capacitance in the coupling condenser C_c, in order to obtain a satisfactory deflection on the vacuum-tube voltmeter.

Under such conditions, the conductive component coupled into the measuring circuit from the source output impedance is an appreciable part of the measured circuit conductance. Since the vacuum-tube voltmeter used in these measurements was a single-scale instrument, and since the source output voltage could not be varied without changing the output impedance, it was found impractical to use the same value of C_c for measurements of conductance with the unknown in and out of circuit. In order to eliminate the resulting variation in coupled-in conductance, the source output voltage was held constant, thereby effectively reducing the source output impedance to zero.

The straight-line plot of the resonance-curve data when the unknown is plugged into circuit is shown in Fig. 12. Over the entire range of the standard condenser used the change of the quantity $\sqrt{(V_r/V)^2-1}$ is only ± 1. From this measurement the deduced values of the series resistance and inductance of the unknown are 302.8 ohms and 15.5 microhenrys, respectively.

The resistor was next measured in conjunction with a series condenser as illustrated in Fig. 9(c). The choice of this method arises from the fact that the effective conductance of a reactance and resistance in series is less than the reciprocal of the resistance. If the value of a large conductance is to be determined, the use of a series condenser will, therefore, reduce the conductive component to be measured. In other words, the effective range of the parallel-resonance methods may be extended to lower values of resistance than can be measured

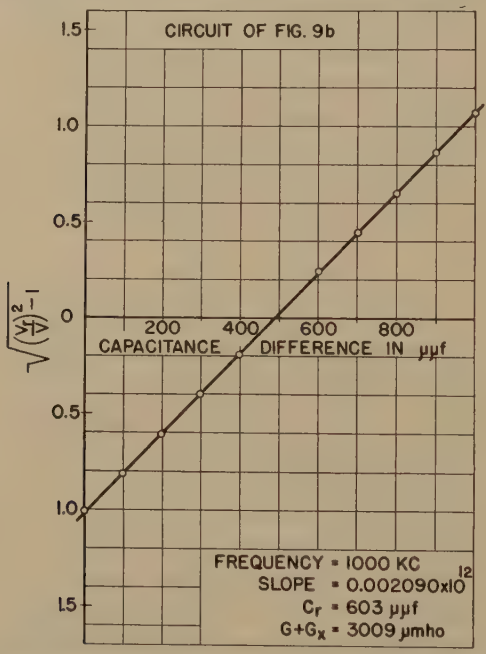

Fig. 12—Plot of resonance-curve data with a 300-ohm resistor connected.

directly. This method of extension is analogous to the method of extending the range of series-resonance methods to higher resistance values than can be measured directly by means of parallel condensers and is subject to the same dangers.[14]

The procedure in this case is first to measure the circuit conductance alone and to determine the resonant capacitance, then to repeat the measurements with the auxiliary condenser C_a in shunt with the measuring circuit, and, finally, to repeat the measurements with the unknown resistor, in series with the auxiliary condenser, in shunt with the measuring circuit.

[14] See, for instance, a discussion of P. B. Taylor's paper by R. F. Field, footnote 3.

The straight-line plot of the resonance-curve data, when the resistor was measured in series with a 200-micromicrofarad auxiliary condenser, is shown in Fig. 13. The resonance curve is seen to be about five times as sharp in this case as it was with the resistor shunted directly across the measuring circuit since there is a total change of 4 in the quantity $\sqrt{(V_r/V)^2-1}$ over about four tenths of the range of the standard condenser used. From the data obtained by this "step-by-step" method the deduced values of the series resistance and inductance of the unknown are 302.3 ohms and 15.3 microhenrys, respectively.

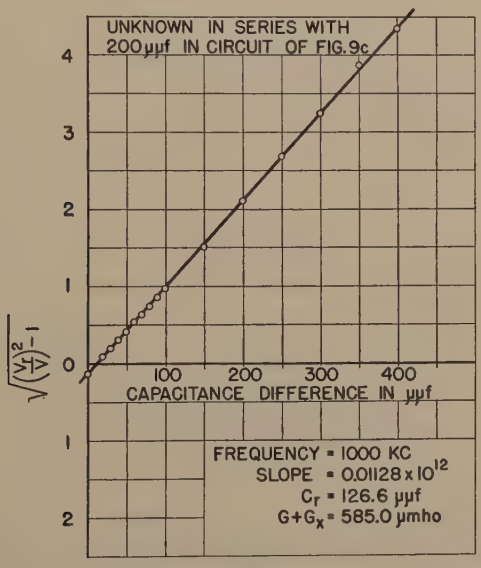

Fig. 13—Plot of resonance-curve data for 300-ohm determination.

The circuit of Fig. 9(d) was used for a third check measurement of the 300-ohm resistor. The method is essentially the same as that outlined above for the step-by-step series condenser method except that the impedance in series with the unknown is the impedance of that portion of the circuit lying to the right of the section *C–C*, rather than that of an auxiliary condenser.

The straight-line plot of the resonance-curve data obtained when the unknown was plugged into circuit is shown in Fig. 14. In this case, it was found that the loss introduced in series with the vacuum-tube voltmeter was so great that, in order to obtain a reasonable voltmeter reading, a very large value of coupling capacitance C_c was necessary. This coupling capacitance effectively shunts the measuring circuit,

as was previously mentioned, and resonance, with the coil used, fell outside the range of the standard condenser when a full-scale deflection of the voltmeter at resonance was obtained. A compromise value of C_c was, therefore, used which gave about half-scale deflection at resonance and a resonance setting just inside the range of the standard condenser. With the reduced voltage at resonance it was not feasible to carry the resonance curve as far down, proportionally, as it was with full-scale

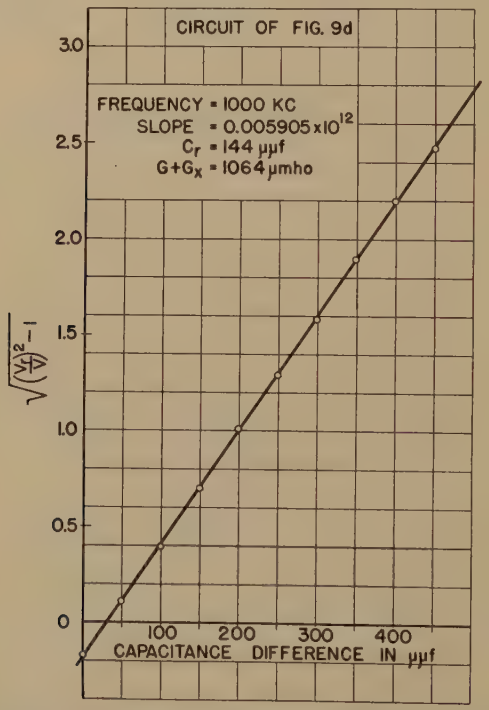

Fig. 14—Plot of resonance-curve data for 300 ohm determination.

deflection at resonance because of $\sqrt{(V_r/V)^2-1}$ and is, accordingly, carried only to 2.5 in Fig. 14. From the data obtained from this method, the deduced values of the series resistance and inductance of the unknown are 302.5 ohms and 14.8 microhenrys, respectively.

The three methods agree within 0.2 per cent at a resistance value of 302.5 ohms. This value indicates an increase in resistance at a frequency of 1 megacycle of about 1.0 per cent over the value of 299.7 ohms measured at direct current. The measured inductance is 15.2 $\pm$0.4 microhenrys.

5. Accuracy Test

The accuracy of the susceptance-variation method was tested by measuring a straight-wire resistance standard, the skin effect of which was less than 0.01 per cent at a frequency of 1 megacycle.

The measurement was made with the circuit of Fig. 9(d).

The value of the resistance deduced from the straight-line plots was 5.543 ohms. This figure checks the value of 5.539 ohms obtained with direct current within 0.1 per cent.

Acknowledgment

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Bibliography

Substitution Method

(1) K. Simons, "Die Dämpfung elektrischer Schwingungen durch eine Funkenstrecke," *Ann. der Phys.*, 4th ser., vol. 13, p. 1044, April 12, (1903).

(2) W. Hahnemann and L. Adelmann, "Verluste in Kondensatoren mit festem Dielektrikum und ihre Dämpfung in Hochfrequenzkreisen," *Elek. Zeit.*, vol. 28, p. 988, October 10, (1907); vol. 28, p. 1010, October 17, (1907).

(3) H. Zahn, "Widerstand von Eisendrähten für schnelle elektrische Schwingungen," *Verh. der Deutsch. Phys. Ges.*, vol. 11, p. 185, April 30, (1909); "Permeabilität des Eisens bei schnellen Schwingungen," *Verh. der Deutsch. Phys. Ges.*, vol. 11, p. 377, September 30, (1909).

(4) C. Fischer, "Strahlung von Antennen," *Phys. Zeit.*, vol. 12, p. 295, April 15, (1911).

(5) L. W. Austin, "A preliminary note on the resistance of radio-telegraphic antennas," *Jour. Wash. Acad. Sci.*, vol. 1, p. 9, July 19, (1911).

(6) L. W. Austin, "Vorläufige Mitteilung über den Widerstand von Antennen für drahtlose Telegraphie," *Phys. Zeit.*, vol. 12, p. 924, November 1, (1911).

(7) H. Jordan, "Zur Kenntnis des Verhaltens anomaler Dielektrika. (Messungen an Papierkondensatoren mit konstanter Spannung und Wechseln sehr verschiedener Frequenz)," *Verh. der Deutsch. Phys. Ges.*, vol. 14, p. 451, April 26, (1912).

(8) L. W. Austin, "Antennen-Widerstand," *Jahr. der draht. Tel. und Tel.*, vol. 5, p. 574, June, (1912); "Antenna resistance," *Bull. Bur. Stand.*, vol. 9, p. 65, March 15, (1913); "Energy losses in some condensers used in high-frequency circuits," *Bull. Bur. of Stand.*, vol. 9, p. 73, March 15, (1913); "The relation between effective resistance and frequency in radio-telegraphic condensers," Proc. I.R.E., vol. 1, pp. 35–39; April, (1913); "Die Energieverluste in einigen in Hochfrequenzkreisen benutzen Kondensatoren," *Jahr. der draht. Tel. und Tel.*, vol. 7, p. 222, June, (1913).

(9) G. E. Bairsto, "On the variation with frequency of the conductivity and dielectric constant of dielectrics for high frequency oscillations," *Proc. Royal Soc.*, ser. A, vol. 96, p. 363, January 2, (1920).

(10) E. Schott, "Hochfrequenzverluste von Gläsern und einigen anderen Dielektricis," *Jahr. der draht. Tel. und Tel.*, vol. 18, p. 82, August, (1921).

(11) H. J. MacLeod, "The variation with frequency of the power loss in dietectrics," *Phys. Rev.*, 2nd ser., vol. 21, p. 53, January, (1923).

(12) P. A. Cooper, "A useful circuit for dielectric constant, power factor and conductivity measurements at high frequencies," *Jour. Sci. Inst.*, vol. 2, p. 342, August, (1925).

(13) E. Möller, "Über die Abhändgigkeit des dielektrischen Verlustwinkels und der Dielektrizitätskonstanten von der Frequenz bei Paraffin, Hexan, Xylol, Quarz, Glas, Porzellan, Hartpapieren, Presspan und einem ionisierten Luftkondensator," *Archiv für Elek.*, vol. 15, p. 16, (1925).

(14) E. Offermann, "Röhren-Messmethode zur Bestimmung der Verluste in Kondensatoren bei Hochfrequenz," *Zeit. für Hochfrequenz.*, vol. 26, p. 152, (1925).

(15) A. Sayce, "A simple method of measuring the capacity and high frequency loss of a condenser," *Jour. Sci. Inst.*, vol. 3, p. 116, January, (1926).

(16) A. E. Maibauer and T. S. Taylor, "The variation of the resistance of condensers with dial setting at radio frequencies," *Phys. Rev.*, 2nd ser., vol. 27, p. 251, February, (1926).

(17) L. B. Root, "A study of coil resistances at 40 meters," *Gen. Radio Exp.*, vol. 1, p. 1, February, (1927).

(18) P. A. Cooper, "A new loss-measuring device and its application to high-frequency measurements," *Phil. Mag.*, 7th ser., vol. 3, p. 625, March, (1927).

(19) L. Stürmer, "Eine neue Methode zur Messung des Dämpfungswiderstandes von Schwingungskreisen," *Jahr. der draht. Tel. und Tel.*, vol. 29, p. 192, June, (1927).

(20) C. T. Burke, "Substitution method for the determination of resistance of inductors and capacitors at radio frequencies," *Trans. A.I.E.E.*, vol. 46, p. 482, (1927).

(21) H. Sack, "Experimentelles zur Dispersion der Leitfähigkeit," *Phys. Zeit.*, vol. 29, p. 627, September 1, (1928).

(22) R. M. Wilmotte, "A quick and sensitive method of measuring condenser losses at radio frequencies," *Jour. Sci. Inst.*, vol. 5, p. 369, December, (1928).

(23) B. Brendel, O. Mittelstaedt, and H. Sack, "Dispersion der elektrolytischen Leitfähigkeit," *Phys. Zeit.*, vol. 30, p. 577, September 15, (1929).

(24) R. M. Wilmotte, "The comparison of the power factors of condensers," *Exp. Wireless and Wireless Eng.*, vol. 6, p. 656, December, (1929).

(25) A. Deubner, "Eine empfindliche Method zur Vergleichung von Elektrolytwiderständen bei Hochfrequenz," *Phys. Zeit.*, vol. 30, p. 946, December 15, (1929).

(26) J. K. Clapp, "Antenna-measuring equipment," Proc. I.R.E., vol. 18, pp. 571–580; April, (1930).

(27) A. Deubner, "Eine Präzisionsmethode zur Vergleichung von Elektrolytwiderständen bei Hochfrequenz," *Ann. der Phys.*, 5th ser., vol. 5, p. 305, June 10, (1930).

(28) H. Dobretsberger, "Untersuchung des Einflusses adsorbierter Gase auf den Hochfrequenzwiderstand eines Platindraht," *Zeit. für Phys.*, vol. 65, p. 334, October 22, (1930).

(29) F. Vecchiacchi, "I triodi e la misura degli angoli di fase dei condensatori con il metodo di sostituzione nei circuiti resonanti," *L'Elettrotecnica*, vol. 19, p. 93, February 5, (1932).

(30) M. Boella, "Misura ad alta frequenza del l'angolo di perdita dei dielettrici," *L'Elettrotecnica*, vol. 19, p. 99, February 5, (1932).

(31) L. Rohde and W. Schlegelmilch, "Verlustwinkelmessung mit Hochfrequenz an Kondensatoren," *Elek. Zeit.*, vol. 54, p. 580, June 15, (1933).

(32) H. C. Forbes and L. J. Gorman, "Skin effect in rectangular conductors," *Elec. Eng.*, vol. 52, p. 636, September, (1933).

(33) S. Helt, "Measuring resistance of broadcast antennas," *Electronics*, vol. 7, p. 45, February, (1934).

(34) E. Müller and O. Zinke, "Grundsätzliches über die Messung von Verlustwinkeln und die Bedeutung der Temperaturabhängigkeit des tgδ in der Hochfrequenztechnik," *Hochfrequenz. und Elektroakustik*, vol. 43, p. 145, May, (1934).

(35) L. Rohde and H. Schwartz, "Verlustwinkelmessung bei 10^8 Hz," *Hochfrequenz. und Elektroakustik.*, vol. 43, p. 156, May, (1934).

(36) M. Boella, "Direct loss-angle measurements in condensers at high frequencies," *Alta Freq.*, vol. 4, p. 5, February, (1935).
(37) L. Rohde, "Dielectric measurements at 60 cm.," *Zeit. für Tech. Phys.*, vol. 16, p. 636, December, (1935).
(38) M. Reed, "The effect of stray capacitances to ground in substitution measurements," *Wireless Eng.*, vol. 13, p. 248, May, (1936).

Resistance-Variation Method

(39) R. Lindemann, "Über Dämpfungsmessungen mittels ungedämpfter elektrischer Schwingungen," *Verh. der Deutsch. Phys. Ges.*, vol. 11, p. 28, January 15, (1909).
(40) A. Meissner, "Über die Konstruktion von Spulen der Hochfrequenztechnik und ihre Verwendung," *Jahr. der draht. Tel. und Tel.*, vol. 3, p. 57, October, (1909).
(41) R. Lindemann, "Über die Widerstandszunahme unterteilter Leiter bei schnellen Schwingungen," *Ver. der Deutsch. Phys. Ges.*, vol. 11, p. 682, November 30, (1909); "Die Energieverluste in mehrlagigen Spulen bei schnellen Schwingungen," *Ver. der Deutsch. Phys. Ges.*, vol. 12, p. 572, July 30, (1910).
(42) J. A. Fleming and G. B. Dyke, "The measurement of energy losses in condensers traversed by high-frequency electric oscillations," *Electrician*, vol. 66, (No. 1707), p. 658, February 3, (1911).
(43) J. A. Fleming and G. B. Dyke, "The measurement of energy losses in condensers traversed by high-frequency oscillations," *Proc. Phys. Soc.* (London), vol. 23, p. 117, (1910–1911).
(44) R. Lindemann, "Untersuchungen über die Widerstandszunahme von Drahtlitzen bei schnellen elektrischen Schwingungen," *Jahr. der draht. Tel. und Tel.*, vol. 4, p. 561, July, (1911).
(45) R. Lindemann and W. Huter, "Über den Skineffekt von Flachspulen und kurzen Zylinderspulen bei schnellen elektrischen Schwingungen," *Verh. der Deutsch. Phys. Ges.*, vol. 15, p. 219, April, (1913).
(46) S. Loewe, "Über die erreichbare Genauigkeit der Widerstandsmessung in Hochfrequenzkreisen," *Jahr. der draht. Tel. und Tel.*, vol. 7, p. 365, September, (1913).
(47) L. Cohen, "A method for determining logarithmic decrements," Proc. I.R.E., vol. 2, pp. 237–252; September, (1914); "Ein Verfahren zur Bestimmung logarithmischer Dekremente," *Jahr. der draht. Tel. und Tel.*, vol. 10, p. 340, February, (1916).
(48) G. Breit, "The distributed capacity of inductance coils," *Phys. Rev.*, 2nd ser., vol. 17, p. 649, June, (1921); "High-frequency resistance of inductance coils," *Bur. Stand. Sci. Paper* No. 430, February 24, (1922).
(49) A. Bailey, "A study of the effect of adsorbed gas on the high-frequency resistance of copper wire," *Phys. Rev.*, 2nd ser., vol. 20, p. 154, February, (1922).
(50) R. Mesny, "Étude sur les résistances à haute fréquence," *L'Onde Elect.*, vol. 1, p. 160, March, (1922); vol. 1, p. 231, April, (1922).
(51) P. O. Pedersen, "A method for the measurement of r. f. resistances," *Wireless World and Radio Rev.*, vol. 10, p. 135, April 29, (1922).
(52) J. H. Morecroft, "Resistance and capacity of coils at radio frequencies," Proc. I.R.E., vol. 10, pp. 261–314; August, (1922).
(53) F. E. Nancarrow and I. J. Cohen, "High-frequency resistance measurements," *P.O.E.E. Jour.*, vol. 16, p. 71, April, (1923).
(54) A. B. Bryan, "Dielectric losses at radio frequencies in liquid dielecrics," *Phys. Rev.*, 2nd ser., vol. 22, p. 399, October, (1923).
(55) R. V. Guthrie, "Electrical constants of dielectrics for radio frequency currents," Proc. I.R.E., vol. 12, p. 841, December, (1924).
(56) N. Nikitin, "Anwendung des Thomsonschen Kreises zur Untersuchung der magnetischen Spektra," *Zeit. für Phys.*, vol. 29, 288, (1924).
(57) L. B. Turner, "The effect of oxidation on the high frequency resistance of aerial wires: with a note on measuring the resistance of thick wires," *Jour. I.E.E.*, (London), vol. 63, p. 149, January, (1925).
(58) C. N. Weyl and S. Harris, "A method of measuring at radio frequencies the equivalent series resistance of condensers intended for use in radio receiving circuits," Proc. I.R.E., vol. 13, pp. 109–150; February, (1925).

(59) P. O. Pedersen, "An electrometer method for the measurement of radio frequency resistance," PROC. I.R.E., vol. 13, pp. 215–244; April, (1925).
(60) R. M. Wilmotte, "Parasitic losses in inductance coils at radio frequencies," *Exp. Wireless and Wireless Eng.*, vol. 2, p. 477, May, (1925).
(61) C. D. Callis, "Measurement of the resistance of a condenser at radio frequency," *Phil. Mag.*, 7th ser., vol. 1, p. 428, February, (1926).
(62) P. K. Turner, "H. f. resistance," *Exp. Wireless and Wireless Eng.*, vol. 3, p. 134, March, (1926).
(63) E. L. Hall, "Resistance of conductors of various types and sizes as windings of single layer coils at 150 to 6000 kilocycles," *Bur. Stand. Tech. Paper* No. 330, October 27, (1926).
(64) W. M. Roberds, "The resistance of copper wires at very high frequencies," *Phys. Rev.*, 2nd ser., vol. 29, p. 165, January, (1927).
(65) S. L. Brown and M. Y. Colby, "Electrical measurements at radio frequencies," *Phys. Rev.*, 2nd ser., vol. 29, p. 717, May, (1927).
(66) S. L. Brown, C. F. Wiebusch, and M. Y. Colby, "The high frequency resistance of a Bureau of Standards type variable air condenser," *Phys. Rev.*, vol. 29, p. 887, June, (1927).
(67) D. W. Dye, "Basic measurements of the effective resistance of condensers at radio frequencies," *Proc. Phys. Soc.* (London), vol. 40, p. 285, (1927–1928).
(68) G. R. Wait, F. G. Brickwedde, and E. L. Hall, "Electrical resistance and magnetic permeability of iron wire at radio frequencies," *Phys. Rev.*, 2nd ser., vol. 31, p. 303, February, (1928).
(69) F. Eisner, H. Fassbender, and G. Kurlbaum, "Leistungs- und Strahlungsmessungen an Flugzeug- und Bodenstationen," *Zeit. für Hochfrequenz.*, vol. 31, p. 109, April, (1928).
(70) W. Jackson, "Dielectric losses in single-layer coils at radio frequencies," *Exp. Wireless and Wireless Eng.*, vol. 5, p. 255, May, (1928).
(71) G. W. Sutton, "A method for the determination of the equivalent resistance of air condensers at high frequencies," *Proc. Phys. Soc.* (London), vol. 41, part 2, p. 126, February 15, (1929).
(72) A. J. Palermo and F. W. Grover, "A study of the high-frequency resistance of single layer coils," PROC. I.R.E., vol. 18, pp. 2041–2058; December, (1930).
(73) W. Jackson, "High frequency resistance measurement by the use of a variable mutual inductance," *Jour. I.E.E.* (London), vol. 68, p. 296, (1930).
(74) B. Brendel, "Die Dispersion der Leitfähigkeit der Elektrolyre," *Phys. Zeit.*, vol. 32, p. 327, April 15, (1931).
(75) W. Jackson, "Losses in liquid dielectrics at radio frequencies," *Wireless Eng. and Exp. Wireless*, vol. 9, p. 14, January, (1932).
(76) P. B. Taylor, "Method for measurement of high resistance at high frequency," PROC. I.R.E., vol. 20, pp. 1802–1806; November, (1932).
(77) A. J. Palermo, "The effect of displacement currents on the high-frequency resistance of circular single-layer coils," PROC. I.R.E., vol. 20, pp. 1807–1810; November, (1932).
(78) E. A. Laport, "Some notes on the influence of stray capacitance upon the accuracy of antenna resistance measurements," PROC. I.R.E., vol. 22, pp. 657–669; May, (1934).
(79) J. G. Chaffee, "The determination of dielectric properties at very high frequencies," PROC. I.R.E., vol. 22, pp. 1009–1020; August, (1934).
(80) O. S. Puckle, "The behavior of high resistances at high frequencies," *Wireless Eng. and Exp. Wireless*, vol. 12, p. 303, June, (1935).
(81) H. Nitschmann, "Über die Messung des Wirkwiderstandes nach der Resonanzmethode," *Hochfrequenz. und Elektroakustik*, vol. 46, p. 91, September, (1935).

REACTANCE-VARIATION METHOD

(82) V. Bjerknes, "Ueber electrische Resonanz," *Wied. Ann.*, vol. 55, p. 121, May, (1895).
(83) S. Lagergren, "Ueber die Dämpfung electrischer Resonatoren," *Wied. Ann.*, vol. 64, p. 290, February 1, (1898).
(84) P. Drude, "Die Dämpfung von Kondensatorkreisen mit Funkenstrecke," *Ann. der Phys.*, 4th ser., vol. 15, p. 709, December 6, (1904).

(85) G. Rempp, "Die Dämpfung von Kondensatorkreisen mit Funkenstrecke," *Ann. der Phys.*, 4th ser., vol. 17, p. 627, August 22, (1905).
(86) T. Noda, "Dämpfung eines Kondensatorkreises mit einem Zusatzkreise," *Ann. der Phys.*, 4th ser., vol. 19, p. 715, March 27, (1906).
(87) H. Brandes, "Über die Dämpfung und Energieausnutzung einiger Senderanordnungen der drahtlosen Telegraphie," *Ann. der Phys.*, 4th ser., vol. 22, p. 640, April 4, (1907).
(88) F. Conrat, "Über Messung des Strahlungsdekrementes stabförmiger Leiter," *Ann. der Phys.*, 4th ser., vol. 22, p. 659, April 4, (1907).
(89) W. Eickhoff, "Über Messung des Funkenwiderstands in Kondensatorkreisen," *Phys. Zeit.*, vol. 8, p. 494, August 1, (1907); "Über Messungen an unterteilten Funkenstrecken," *Phys. Zeit.*, vol. 8, p. 497, August 1, (1907).
(90) R. von Traubenberg and W. Hahnemann, "Untersuchungen der Dämpfung von Funkenstrecken in Schwingungskreisen der Hochfrequenztechnik," *Phys. Zeit.*, vol. 8, p. 498, August 1, (1907).
(91) M. Wien, "Eine Fehlerquelle bei der Messung der Dämpfung elek, trischer Schwingungen nach der Bjerknesschen Methode," *Phys. Zeit.*, vol. 8-p. 764, October 24, (1907); also, *Verh. der Deutsch. Phys. Ges.*, vol. 5, p. 478, October 30, (1907).
(92) R. von Traubenberg and B. Monasch, "Über die Verwendung kontinuierlicher elektromagnetischer Schwingungen bei Dämpfungsmessungen," *Phys. Zeit.*, vol. 8, p. 925, December 15, (1907).
(93) A. Jollos, "Der Funkenwiderstand in Kondensatorkreisen," *Jahr. der draht. Tel. und Tel.*, vol. 1, p. 227, (1907).
(94) M. Wien, "Über die Rückwirkung des Messkreises bei der Bestimmung der Dämpfung elektrischer Schwingungen nach der Bjerknesschen Methode," *Jahr. der draht. Tel. und Tel.*, vol. 1, p. 461, (1907).
(95) R. H. Rendahl, "Über die Verwendung kontinuierlicher elektromagnetischer Schwingungen bei Dämpfungsmessungen," *Phys. Zeit.*, vol. 9, p. 135, February 15, (1908).
(96) M. Wien, "Über die Dämpfung von Kondensatorschwingungen," *Ann. der Phys.*, 4th ser., vol. 25, p. 625, March 31, (1908).
(97) B. Macků, "Über die Bestimmung der Dämpfung von Kondensatorschwingungen bei beliebig enger induktiver Koppelung," *Phys. Zeit.*, vol. 9, p. 437, July 1, (1908).
(98) C. Fischer, "Dekrementbestimmung mit kontinuierlichen elektrischen Wellen," *Ann. der Phys.*, 4th ser., vol. 28, p. 57, December 28, (1908).
(99) B. Macků, "Theorie zweier gekoppelter Oszillationskreise unter besonderer Berucksichtigung der Dämpfungsmessungen," *Jahr. der draht. Tel. und Tel.*, vol. 2, p. 251, February, (1909).
(100) W. Hahnemann, "Die Messmethoden, Grosse und Bedeutung der Dämpfung in der drahtlosen Telegraphie," *Jahr. der draht, Tel. und Tel.*, vol. 2, p. 293, February, (1909); vol. 2, p. 477, June, (1909).
(101) M. Wien, "Über die Dämpfung von Kondensatorschwingungen," *Ann. der Phys.*, 4th ser., vol. 29, p. 679, July, (1909).
(102) C. Fischer, "Strahlungsdekrement, wirksame Kapazität und Selbstinduktion einer Antenne," *Ann. der Phys.*, 4th ser., vol. 32, p. 799, July 8, (1910).
(103) L. Mandelstam and N. Papalexi, "Über eine Methode zur Messung von Logarithmischen Dekrementen und Schwingungszahlen elektromagnetischer Schwingungssystem," *Ann. der Phys.*, 4th ser., vol. 33, p. 490, September 22, (1910).
(104) W. F. Zorn, "Über die Abhängigkeit der Dämpfung in Kondensatorkreisen mit Funkenstrecke von der Gestalt und dem Material der Elektroden sowie von dem Dielektrikum in der Funkenstrecke," *Jahr. der draht. Tel. und Tel.*, vol. 4, p. 260, February, (1911); vol. 4, p. 382, April, (1911).
(105) B. Macků, "Über den Einfluss des fruhzeitigen Ausloschens des Funkens auf Dämpfungsmessungen," *Ann. der Phys.* 4th ser., vol. 34, p. 941, May 9, (1911).
(106) L. Mandelstam and N. Papalexi, "Uber eine Methode zur Messung von logarithmischen Dekrementen und Schwingungszahlen elektromagnetischer Shwingungssystem," *Jahr. der draht. Tel. und Tel.*, vol. 4, p. 605, July, (1911).

(107) H. Riegger, "Über gekoppelte Kondensatorkreise bei sehr kurzer Funkenstrecke," *Jahr. der draht. Tel. und Tel.*, vol. 5, p. 35, September, (1911).

(108) P. Ludewig, "Ein Dämpfungsmesser fur die Praxis der drahtlosen Telegraphie," *Phys. Zeit.*, vol. 12, p. 763, September 15, (1911).

(109) S. Loewe, "Eine Ursache storender Induktion bei Messungen in Hochfrequenzkreisen," *Jahrb. der draht. Tel. und Tel.*, vol. 6, p. 325, January, (1913).

(110) F. Kiebitz, "Zur Theorie der Dämpfungsmessungen elektrischer Schwingungskreise," *Jahr. der draht. Tel. und Tel.*, vol. 6, p. 547, March, (1913).

(111) L. Högelsberger, "Eine Vereinfachung der Methode zur Bestimmung des wirksamen Widerstandes von Schwingungskreisen mit Hilfe gedämpfter Schwingungen," *Verh. der Deutsch. Phys. Ges.*, vol. 15, p. 167, March 15, (1913); also, *Jahr. der draht. Tel. und Tel.*, vol. 7, p. 182, June, (1913).

(112) F. A. Kolster, "A direct-reading instrument for measuring the logarithmic decrement and wave length of electromagnetic waves," *Jour. Wash. Acad. Sci.*, vol. 4, p. 569, December 4, (1914); "A direct-reading decremeter and wave meter," Proc. I.R.E., vol. 3, pp. 29–54; March, (1915); "A direct-reading instrument for measuring the logarithmic decrement and wave length of electromagnetic waves," *Bull. Bur. Stand.*, vol. 11, p. 421, May 10, (1915); "Ein direkt anzeigender Dekrement- und Wellenmesser," *Jahr. der draht. Tel. und Tel.*, vol. 10, p. 316, February, (1916).

(113) W. Burstyn, "Einfache Berechnung der Formeln für die Dämpfung eines Schwingungskreises und ihre Messung nach Bjerknes," *Jahr. der draht. Tel. und Tel.*, vol. 10, p. 347, February, (1916).

(114) H. Wiesent, "Über einen direkt zeigender Dämpfungsmesser," *Jahr. der draht. Tel. und Tel.*, vol. 12, p. 330, December, (1917).

(115) E. Giebe and E. Alberti, "Störende Kapazitätswirkungen bei Hochfrequenzmessanordnung," *Jahr. der draht. Tel. und Tel.*, vol. 16, p. 242, October, (1920).

(116) E. Alberti, "Anormale Resonanzkurven, die durch Summation zweier phasanverschobener Ströme antstehen," *Jahr. der draht. Tel. und Tel.*, vol. 16, p. 252, October, (1920).

(117) H. Pauli, "Zur Theorie der Messungen mittels ungedämpfter elektrischer Schwingungen an zwei magnetisch gekoppelten Kreisen," *Jahr. der draht. Tel. und Tel.*, vol. 17, p. 322, May, (1921); also, *Ann. der Phys.*, 4th ser., vol. 65, p. 274, June 14, (1921); "Dämpfungsmessungen mit ungedämpften elektrischen Schwingungen," *Zeit. für Phys.*, vol. 5, p. 376, (1921); "Ein neues Verfahren zur Dämpfungsmessungen mit elektrischen Schwingungen," *Zeit. für Phys.*, vol. 6, p. 118, (1921).

(118) J. Tykocinski-Tykociner, and R. W. Armstrong, "Determination of frequency and decrement by means of plate current variations of thermionic oscillator tubes," *Phys. Rev.*, 2nd ser., vol. 33, p. 634, April, (1929).

(119) H. Kuhlewein, "Messung der Dielektrizitätskonstanten und der scheinbaren Leitfähigkeit von Isolierstoffen bei Hochfrequenz," *Zeit. für Tech. Phys.*, vol. 10, p. 280, July, (1929).

(120) E. B. Moullin, "Notes on the detuning method of measuring the high-frequency resistance of a circuit," *Exp. Wireless and Wireless Eng.*, vol. 7, p. 367, July, (1930).

(121) J. Tykocinski-Tykociner, "Determination of Frequency and Damping of Resonating Circuits," *Phys. Rev.*, 2nd ser., vol. 37, p. 461, February 15, (1931).

(122) W. Anderson, "The dielectric constant and power factor of some solid dielectrics at radio frequencies," *Phil. Mag.*, 7th ser., vol. 13, p. 986, May, (1932).

(123) S. K. Mitra and B. C. Sil, "On the variation of the resistance of thermionic valves at high frequencies," *Phil. Mag.*, 7th ser., vol. 13, p. 1081, June, (1932).

(124) E. B. Moullin, "A method of measuring the effective resistance of a condenser at radio frequencies, and of measuring the resistance of long straight wires," *Proc. Royal Soc.*, ser. A, vol. 137, p. 116, July 1, (1932).

(125) H. Schaefer, "Absolutmessung der Hochfrequenzleitfähigkeit von Flussigkeiten bei 3 m Wellenlänge," *Zeit. für Phys.*, vol. 77, p. 117, July 19, (1932).

(126) V. Petržílka and W. Fehr, "Über ein Verfahren zur Dämpfungsmessung und Schwingungskreisen," *Zeit. für Tech. Phys.*, vol. 13, p. 472, October, (1932).

(127) W. Jackson, "The analysis of air condenser loss resistance," Proc. I.R.E., vol. 22, pp. 957–963; August, (1934). "Skin effect in rectangular conductors at high frequencies," *Phil. Mag.*, 7th ser., vol. 18, p. 433, September, (1934).

(128) H. Kessler, "Messungen an festen technischen Isolierstoffen bei 3.10^6–7, 5.10^7 Hertz," *Hochfrequenz. und Elektroakustik.*, vol. 45, p. 91, March, (1935).

(129) Richard Günther, "Über eine Messung der Elektrischen Ersatzgrössen von Piezo-Elektrischen Kristallen," *Hochfrequenz. und Elektroakustik.*, vol. 45, p. 185, June, (1935).

(130) O. Schütte and G. Weiss, "Damping measurement," *Elek. Nach. Tech.*, vol. 12, p. 204, July, (1935).

(131) M. J. O. Strutt and A. Van der Ziel, "Messungen der Charakteristischen Eigenschaften von Hochfrequenz-Empfangsröhren Zwischen 1, 5 und 60 Megahertz," *Elek. Nach. Tech.*, vol. 12, p. 347, November, (1935).

(132) M. Boella, "Un metodo assoluto per la misura della resistenza equivalente dei circuiti oscillatori," *Alta Frequenza*, vol. 4, December, (1935).

(133) E. Fromy, "Montage d'essai pour les Mesures aux Fréquences Radioélectriques," *Rev. Gen. de l'Elec.*, vol. 34, p. 611, April 25, (1936).

(134) M. Beauvilain, "La resonance de tension et ses applications," *l'Onde Élec.*, vol. 15, p. 650, October, (1936).

(135) L. Hartshorn and W. H. Ward, "The measurement of the permittivity and power factor of dielectrics at frequencies from 10^4 to 10^8 cycles per second," *Proc. Wireless Section I.E.E.* (London), vol. 12, p. 6, March, (1937).

General

(136) K. Küpfmüller, "Messungen bei sehr hohen Frequenzen," *Zeit. für Tech. Phys.*, vol. 14, p. 447, November, (1933).

(137) R. M. Davies, "On the avoidance of errors due to the "leads" in radio-frequency measurements," *Phil. Mag.*, ser. 7, vol. 20, p. 75, July, (1935).

(138) H. Schwarz, "Verlustmessung an Spulen bei Hochfrequenz," *Hochfrequenz. und Elektroakustik.*, vol. 46, p. 50, August, (1935).

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RECTANGULAR HOLLOW-PIPE RADIATORS*

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Summary—*The open hollow pipe of rectangular cross section is discussed from both practical and theoretical standpoints as a radiator and absorber of ultra-short electromagnetic waves. A theory of operation is derived and it is verified by a series of measured radiation patterns taken at wavelengths of 50 to 100 centimeters.*

Introduction

THIS paper reports some of the results of a theoretical and experimental investigation of the radiation properties of open-ended rectangular conducting pipes. Theoretical calculations of the radiation patterns have been made, and a series of patterns have been measured at wavelengths between 50 and 100 centimeters to determine the actual patterns that may be obtained.

The many desirable features possessed by the hollow pipe as a conductor for electromagnetic energy at extremely high frequencies, such as the substantially perfect shielding, the low dielectric and conductive losses, and the simple and rugged construction, suggested that an open-ended hollow pipe might serve as an effective radiator or absorber for radio waves, in place of the more conventional types of antennas and reflectors.[1,2]

Representative radiation patterns for two types of waves in circular pipes (the E_0 and the H_1 waves, respectively) have already been published.[2] A short circular metal cylinder with an antenna along the axis has been suggested as radiating means.[3] Flared horns attached to a pipe have also been described.[1,2] A separate investigation of hollow pipes of different cross-sectional shapes[4] has shown the wisdom of using

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[1] G. C. Southworth, "Hyper-frequency wave guides—general considerations and experimental results," *Bell Sys. Tech. Jour.*, vol. 15, pp. 284–309; April, (1936).

[2] W. L. Barrow, "Transmission of electromagnetic waves in hollow tubes of metal," Proc. I.R.E., vol. 24, pp. 1298–1328; October, (1936).

[3] L. Bergmann and L. Kruegel, "Messungen im Strahlungsfeld einer im Innern eines metalischen Hohlzylinders erregten Linearantenne," *Ann. der Phys.*, vol. 21, pp. 113–138; October, (1934).

[4] L. J. Chu and W. L. Barrow, "Electromagnetic waves in hollow metal tubes of rectangular cross section," Proc. I.R.E., this issue, pp. 1520–1555.

pipes of *rectangular* cross section and *H* waves, rather than those of circular cross section proposed heretofore, because it is only in rectangular pipes that a hollow-pipe wave can be obtained with a configuration ideally adapted to the production of strictly linearly polarized radio beams.

General Considerations

The radiating or absorbing means that will be discussed comprises, in its simplest form (Fig. 1), a hollow conducting pipe open at one end and joined at the other end to a terminal device for connecting the hollow pipe with a conventional two-conductor circuit. The terminal

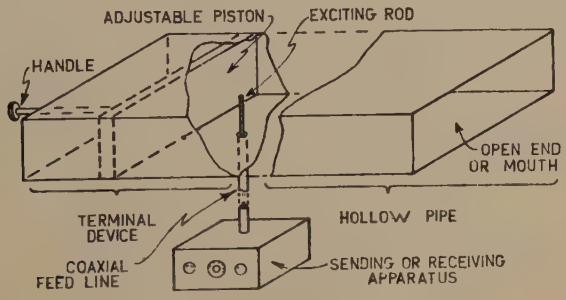

Fig. 1—Open-end hollow-pipe radiator.

illustrated has a vertical exciting rod, an adjustable piston reflector, and a coaxial-line connection between the rod and the sending or receiving apparatus. If electromagnetic energy at a frequency above the critical value of the pipe is delivered to the exciting rod from a transmitting apparatus, it will be transmitted down the pipe as hollow-pipe waves to the open end, where a part of it will be radiated into space, the remainder being reflected back into the pipe. The efficiency of transmission through the pipe and of the radiation from its end is high and the radiant energy is propagated predominantly in the forward direction. In this way, an open hollow pipe may be used as a distinctive type of transmitting antenna. In a reverse manner, when a radio wave of frequency greater than the critical value impinges on the open end of the pipe, a part of the energy will be propagated through the interior of the pipe to the terminal device and thence to the receiving apparatus where it may be demodulated. Therefore, an open hollow pipe may be used as a receiving antenna that is responsive to waves whose frequency exceeds the critical value.

The ease with which the hollow-pipe radiator can be put in operation and maintained is striking. Usually, the only important adjust-

ment is the distance between the plunger or closing wall and the exciting rod, but this adjustment can be made in several minutes even with rudimentary equipment. Furthermore, this adjustment is not too critical and the system operates well over a relatively wide range of frequencies. By replacing the plunger with a dissipative medium or a nonreflective termination, this broad-band feature can be extended. The facility with which different degrees of sharpness in the two planes of the beam may be produced by a suitable choice of cross section is also considered a practical advantage.

When the simplest wave, the $H_{0,1}$ wave, is used in a rectangular pipe with a vertical exciting rod, as in Fig. 1, the radiated wave will be vertically polarized. A vertically polarized receiving antenna would then be used, and, since such antennas generally receive waves of only one polarization, all of the transmitted energy is utilized in the most effective way. This simple polarization is a feature peculiar to the rec-

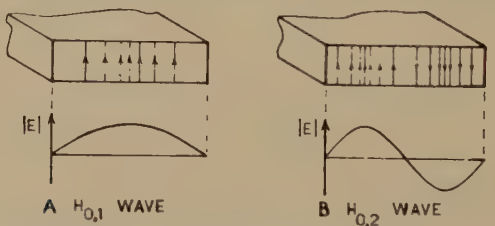

Fig. 2—Electric-intensity distribution and configuration across the mouth of a pipe for two types of waves in a rectangular pipe.

tangular pipe and it is the principle reason for preferring this pipe to circular and other shapes. The electric field intensity across the mouth of the pipe for the $H_{0,1}$ wave, shown in Fig. 2A, is strictly vertical and has a half-sinusoid distribution in the horizontal direction with the maximum at the center of the pipe, and a uniform distribution in the vertical direction. When such a pipe and terminal device are used as a receiving antenna, it responds only to vertically polarized waves. The radiation patterns and the comments to follow apply equally well to the transmitting or to the receiving applications of the open hollow pipe.

The distribution of the next higher-order wave, the $H_{0,2}$ wave is shown in Fig. 2B; it has a full-period sinusoidal distribution. Whereas the $H_{0,1}$ wave radiates a single beam directly to the front, the $H_{0,2}$ wave radiates two sidewise beams. In general, the H-zero-odd-integer waves have a principal forward beam, but the H-zero-even-integer waves have no forward beam but two principle beams arranged sym-

metrically one on each side. Although any of the other types of waves may be used, viz., the $H_{n,m}$ or the $E_{n,m}$ waves, only the $H_{0,m}$ and particularly the $H_{0,1}$ types appear to have characteristics adapted to the usual radiation problems, and the remaining discussion will deal almost exclusively with these latter types.

Theoretical Calculation of Radiation Patterns

An exact mathematical solution of this problem is not possible because of our inability to satisfy the boundary conditions on a finite open-ended pipe. Nevertheless, an approximate result has been obtained that is sufficiently accurate for most practical purposes. The details of the analysis are given separately in the Appendix; the body of the paper will deal with the theoretical results in an interpretive way. The radiation patterns for the $H_{0,m}$ waves only will be discussed. A summary of the method follows.

The components of electric intensity E_y and of magnetic intensity H_x and H_z for an $H_{0,m}$ wave inside an infinitely long rectangular pipe are given by[4]

$$\left.\begin{aligned} E_y &= C_1\left(\frac{\omega}{c}\right)^2 \left\{\begin{matrix}\cos\left(\frac{m\pi}{b}z\right)\\ \sin\left(\frac{m\pi}{b}z\right)\end{matrix}\right\} e^{-i\beta x+i\omega t} \\ H_x &= -C_1 i\omega\epsilon\left(\frac{m\pi}{b}\right)\left\{\begin{matrix}\sin\left(\frac{m\pi}{b}z\right)\\ \cos\left(\frac{m\pi}{b}z\right)\end{matrix}\right\} e^{-i\beta x+i\omega t} \\ H_z &= C_1\omega\epsilon\beta\left\{\begin{matrix}\cos\left(\frac{m\pi}{b}z\right)\\ \sin\left(\frac{m\pi}{b}z\right)\end{matrix}\right\} e^{-i\beta x+i\omega t} \end{aligned}\right\} \quad (1)$$

where m denotes the number of half sinusoids or maxima in the horizontal distribution. When m is an odd integer, the upper of the two functions in brackets is chosen, and when even the lower is taken. The quantity C_1 is a constant, b is the width of the pipe in the z direction, $\beta=\sqrt{(2\pi/\lambda)^2-(m\pi/b)^2}$, $i=\sqrt{-1}$, t=time in seconds, and $\omega=2\pi\cdot$frequency. Although this distribution is derived for an infinitely long pipe, a number of experimental measurements have established that it also holds to a good degree of approximation in the plane of the open end of

a pipe of finite length. The distribution (1) is therefore assumed to represent the field at the mouth of the radiator. The Hertzian vector, Π, from which (1) may be derived, is also known at the mouth. Huygens' principle is next employed to calculate the Hertzian vector Π' for the space outside the pipe at a distance from the mouth large compared to the wavelength λ and the dimensions a, b. The several components of electric and magnetic intensity in the spherical co-ordinate system of Fig. 3 are then derived from Π'. The radiation

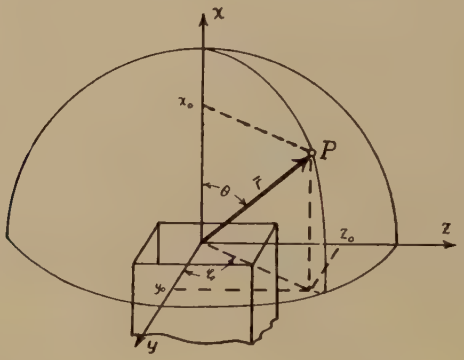

Fig. 3—Polar co-ordinate system in relation to the open end of the pipe.

field thus obtained has only transverse components and the absolute magnitude is the quantity usually measured experimentally. A curve of the absolute magnitude of the electric intensity versus the space angles θ, ζ comprises the usual field pattern or radiation characteristic of the system.

We shall make use of the abbreviations

$$\left.\begin{aligned} A &= \frac{\pi a}{\lambda}\sin\theta = \pi W_v \sin\theta \\ B &= \frac{\pi b}{\lambda}\sin\theta = \pi W_h \sin\theta \\ M &= \frac{\beta c}{\omega} = \sqrt{1-\left(\frac{m\lambda}{2b}\right)^2} \end{aligned}\right\} \tag{2}$$

and the designations

$$\left.\begin{aligned} W_v &= \text{vertical aperture} = \frac{a}{\lambda} \\ W_h &= \text{horizontal aperture} = \frac{b}{\lambda}. \end{aligned}\right\} \tag{3}$$

The functions of principal interest are those giving the radiation patterns in the x, y plane (vertical characteristic) and in the x, z plane (horizontal characteristic); they are:

In the x, y plane (vertical characteristic)

$$E_{\theta m}(\theta, 0) = C\left|(\cos\theta + M)\left[\frac{\sin A}{A}\right]\cos\theta\sin\left(\frac{m\pi}{2}\right)\right|. \tag{4}$$

In the x, z plane (horizontal characteristic)

$$E_{\xi m}\left(\theta, \frac{\pi}{2}\right) = C\left|(\cos\theta + M)\left[\frac{\sin\left(B + \frac{m\pi}{2}\right)}{B^2 - \left(\frac{m\pi}{2}\right)^2}\right]\right|. \tag{5}$$

$E_{\theta m}(\theta, 0)$ gives the relative magnitude of the electric intensity transverse to the radius vector r on a circle in the x, y plane of radius large compared to the wavelength and to a and b. $E_{\xi m}$ gives the corresponding quantity in the x, z plane. The subscript m denotes the order of the $H_{0,m}$ wave excited in the pipe. The vertical bars indicate that the absolute magnitude of the expression is to be taken.

The first factor C is a constant which depends on the current strength in the exciting rod, the size of the pipe, the frequency of excitation, on the fixed distance at which the field is investigated and on other constant quantities that do not affect the field distribution. The second factor comes from Huygens' principle and takes account of the fact that a wave crosses the plane of the opening (as distinguished from a distribution of current elements in this plane). For $m=1$ and horizontal apertures $W_h>1$, this factor is almost constant ($\leqq 2$) for values of θ less than about ± 30 degrees, hence it has but little influence when the dimensions of the pipe are sufficiently large for the beam to be confined mainly within this limit. The third and fourth factors in (4) and the third factor in (5) are the most important ones in determining the shape of the radiation patterns when the pipe dimensions are several times greater than the wavelength.

The vertical characteristic (4) is zero for $m=$an even integer. For odd integers, it has the form $(\cos\theta+M)\cdot(\sin A)/A\cdot\cos\theta$. This expression has the same form as the corresponding expression for the diffraction of light from a rectangular slit and extensive numerical tables are available.[5]

In order to attach a quantitative measure to the sharpness of the beam we define the following two terms:

[5] Jahnke-Emde, "Tables of Functions," second edition, chapter V.

θ_v = vertical beam angle = the angle included between the first zeros on either side of the center of the vertical characteristic.

θ_h = horizontal beam angle = the angle included between the first zeros on either side of the center of the horizontal characteristic.

The expression for θ_v is readily obtained from (4) and that for θ_h for the special case $m=1$ is also readily found from (5):

$$\theta_v = 2\sin^{-1}\frac{\lambda}{a} = 2\sin^{-1}\left(\frac{1}{W_v}\right)$$
$$\theta_h = 2\sin^{-1}\frac{3\lambda}{2b} = 2\sin^{-1}\left(\frac{3}{2W_h}\right). \tag{6}$$

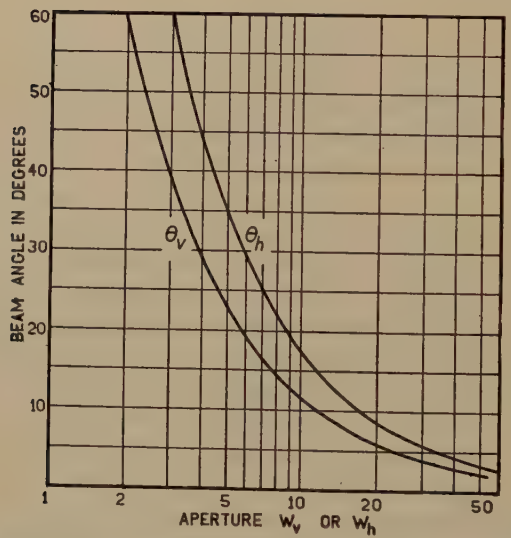

Fig. 4—Horizontal (θ_h) and vertical (θ_v) beam angles versus the apertures W_h and W_v.

These functions are plotted as curves in Fig. 4. For equal apertures, the beam will always be sharper in the vertical plane than in the horizontal plane by a ratio of 2:3. Or, if equal beam angles are desired, the pipe must be made wider in the z direction than in the y direction; the ratio is $b/a=3/2$. The explanation for this difference lies in the distribution of electric intensity across the mouth of the pipe, which is uniform in the y direction and sinusoidal in the z direction. By changing the relative magnitudes of a and b, a beam having given angles in the two planes can be produced. For example, if the b side of the pipe is made much broader than the a side, a beam that is sharp in the

horizontal plane and broad in the vertical plane will result. If it is desired to concentrate the radiation into a sharp pencil-like beam, it is necessary to make both apertures W_v and W_h large compared to unity.

The horizontal characteristic (5) may also be written in the forms

$$\left.\begin{aligned} E_{fm}\left(\theta, \frac{\pi}{2}\right) &= C\left|(\cos\theta + M)\left[\frac{\cos B}{B^2 - \left(\frac{m\pi}{2}\right)^2}\right]\right|, \quad m \text{ odd} \\ E_{fm}\left(\theta, \frac{\pi}{2}\right) &= C\left|(\cos\theta + M)\left[\frac{\sin B}{B^2 - \left(\frac{m\pi}{2}\right)^2}\right]\right|, \quad m \text{ even} \end{aligned}\right\}. \tag{7}$$

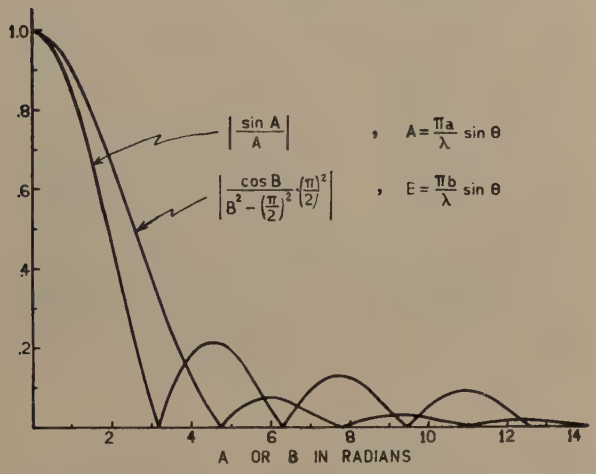

Fig. 5—Principal radiation functions for horizontal and vertical patterns.

The limiting cases in which the pipe dimensions are made to approach their smallest workable values are of interest:

Vertical characteristic

$$\lim_{W_v \to 0} E_{\theta 1}(\theta, 0) = C\,|\,(\cos\theta + M)\cos\theta\,|. \tag{8}$$

Horizontal characteristic

$$\lim_{W_h \to \frac{1}{2}} E_{f1}\left(\theta, \frac{\pi}{2}\right) = C\left|\frac{\cos\left(\frac{\pi}{2}\sin\theta\right)}{\cos\theta}\right|. \tag{9}$$

At the critical frequency, (8) becomes simply $\cos^2\theta$ for the vertical radiation pattern. The limiting case in the horizontal plane (9) is almost a circle, but is somewhat drawn out in the forward direction.

The bracketed functions for the vertical characteristic (4) and the horizontal characteristic (5) for the special case $m=1$ ($H_{0,1}$ waves) are shown in curve form in Fig. 5. Both are reduced to a maximum amplitude of unity. These functions, as was pointed out, are mainly responsible for the shape of the radiation patterns of the open hollow pipe in the two planes when the apertures are greater in magnitude than two or three. The patterns for pipes of different values of a and b are all derivable from these basic curves. An important result is that the relative magnitudes of the secondary lobes of the pattern in either

Fig. 6—Photograph showing the oscillator, coaxial-line connector, and closed sending end of the hollow pipe.

plane compared to the magnitudes of the principle beam remain the same as the beam angle is decreased by making the apertures larger. We observe also that the secondary lobes in the vertical plane are considerably larger in magnitude than those in the horizontal plane, a feature that partially negatives the favorable beam-angle size. The relative magnitudes of the secondary lobes appear quite different, however, if the square of the field intensity is plotted, corresponding to reception with a square-law detector; in this case, the secondary lobes in the horizontal plane are negligible, but those in the vertical plane are objectionably large.

Experimental Apparatus

The experimental results to be presented here comprise a series of measured radiation patterns of a rectangular open hollow pipe, similar to Fig. 1. The pipe was constructed of galvanized iron sheeting with

inside dimensions $a=15$ centimeters, $b=50$ centimeters in sections 239 centimeters long. One or more sections were supported three feet above the ground in a large level open field; the nearest obstruction in the forward 180-degree sector was a fence off to the side at about 700 feet from the pipe. The oscillator was also supported above ground a few feet to the rear of the terminal end of the pipe with a coaxial-line connection between the oscillator and the pipe. The experimental setup is shown in the photograph of Fig. 6.

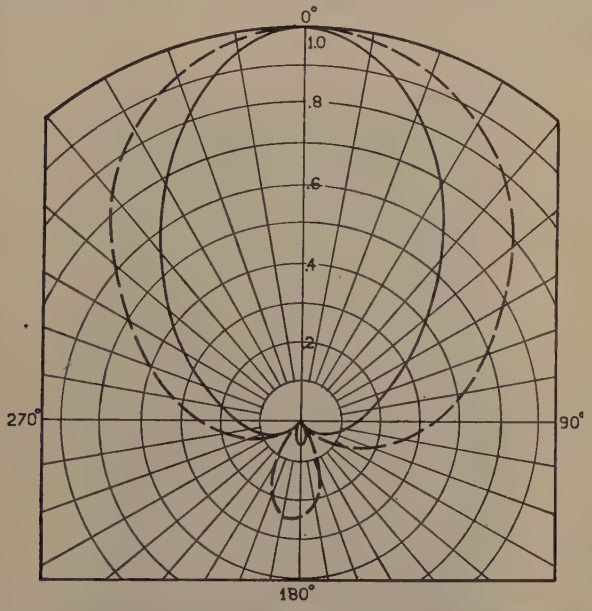

Fig. 7—Directive response characteristics of the parabolic-reflector receiver. Solid curve $\lambda=52$ centimeters, dotted curve $\lambda=90$ centimeters.

The oscillator has been described elsewhere.[6] A voltage regulator on all power supplies was used to prevent line fluctuation from influencing the measurements. The oscillator was plate-circuit modulated at 60 cycles.

The receiver comprises a rod antenna with a parabolic reflector, a type 955 "acorn" triode as a grid-leak detector followed by a calibrated attenuator, an audio-frequency amplifier, and a copper-oxide-type meter. A diminutive tuned circuit in the grid of the detector tube was necessary to prevent stray pickup of local broadcast transmitters. This detector circuit was selected after a comparative investigation of

[6] W. L. Barrow, "An oscillator for ultra-high frequencies," *Rev. Sci. Instr.*, vol. 9, pp. 170–174; June, (1938).

other circuits, tubes and crystals, and modulated and unmodulated signals; its sensitivity and reliability were high compared to those of the other combinations tried.

The rod antenna of adjustable length was provided with a small cylindrical parabolic reflector about 20 inches square to increase the sensitivity and, particularly, to lessen the effect of reflections from the operator during the course of the measurements. The measured directive characteristic of this receiver in the horizontal plane (antenna vertical) at $\lambda=90$ and 52 centimeters are reproduced in Fig. 7. The response at the angles 120 to 160 degrees is very small and the operator

Fig. 8—Photograph of the field-intensity measuring equipment and pushcart.

stands in this region during the readings; in the forward direction the response is not so sharp as to make aiming a problem. As shown in Fig. 8, the entire receiver is mounted in a sturdy pushcarriage and, generally, one person pushes the receiver about the radiator on a large circle marked on the ground, while an assistant, following at some distance, records the data, but one person can operate the entire equipment alone when necessary. Usually, a circle of 100-foot radius was followed, as measurements at greater distances had proved that nothing was to be gained by going farther out. Test measurements showed definitely that the nature of the ground was satisfactorily uniform and

flat at these frequencies for radiation pattern measurements to be made.

The entire apparatus operated remarkably well and the same field-intensity readings could be observed unchanged for periods of hours. A pattern taken on one day could be repeated in detail on another. Calibrations of the receiver at one megacycle showed a 2.17-law response, but in plotting the curves of field intensity, the square roots of the meter readings were taken for the sake of convenience; the error incurred by this simplification is believed compatible with the nature of the measurements. In presenting the experimental data, curves have been drawn rather than points, because of the large number of points that were taken on each pattern.

Measured Radiation Patterns

Representative radiation patterns for the above-described pipe with the $H_{0,1}$ wave are reproduced as solid-line curves in Fig. 9 A, B, C, D and Fig. 10 A, B and a number of calculated values are recorded in the figures as small circles. In all figures, the open end of the pipe is aimed along the 0-degree line; the origin is taken at the center of the mouth of the pipe. The series of curves in Fig. 9 shows the horizontal patterns at four different frequencies for a pipe length $L=4.78$ meters. The horizontal pattern of Fig. 10A and the vertical pattern of Fig. 10B are for a shorter length of pipe, $L=2.39$ meters. The vertical pattern was obtained by turning the pipe on edge with the exciting rod horizontal by turning the receiving antenna into the horizontal plane, and by following the same circle on the ground as before. The receiving antenna was always oriented so as to be at right angles to a line connecting the receiver and the pipe mouth.

Perhaps the most unexpected part of these diagrams is the radiation backwards; i.e., in the sector 90, 180, and 270 degrees. The theory developed above is not valid here, and no calculations of this portion of the diagrams have been made. Direct radiation from the oscillator, coaxial line, or power supply cannot be the cause; because, when the mouth of the pipe was closed with sheet metal, no field could be measured with the receiver as close as several feet from the pipe or oscillator. This back radiation is attributed to diffraction around the mouth of the pipe, particularly the top and bottom edges. In this way, it is believed, currents are set up on the outside of the pipe and radiate in a noncalculable manner. Near the critical frequency, the back radiation is almost as large as the forward radiation, but as the frequency is increased it becomes smaller. For large apertures, particularly when both W_v and W_h are large, it is certain that it will become negligible.

From the four diagrams of Fig. 9, the beam is seen to sharpen appreciably with increasing frequency. The agreement between theory and experiment is considered excellent at $\lambda = 98$ and 52 centimeters and fairly good at 90 and 72 centimeters. In all cases, the measured beam is somewhat sharper than the calculations predict. Other data

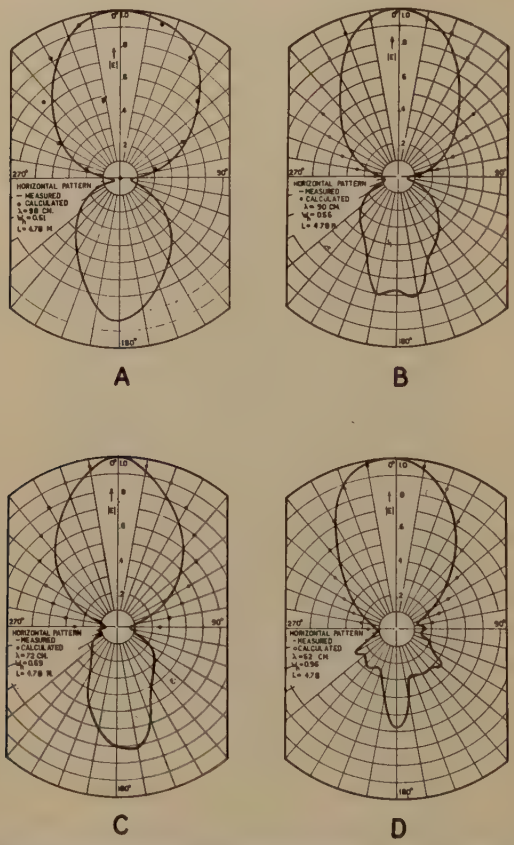

Fig. 9—Measured radiation patterns (solid curve) and calculated values (small circles) in the horizontal plane.

are on hand, to be reported later, indicating that an even better agreement is found with larger apertures. It appears, therefore, that the theory is satisfactorily established and may be relied upon for any practical purpose.

Our experience with directive arrays of half-wave antennas at $\lambda = 50$ centimeters, where stray radiation and reflection from supports, insulators, transmission lines, etc., are impossible to avoid, has gener-

ally shown a comparatively poor agreement between theory and experiment. The open hollow-pipe radiator, on the other hand, is particularly free from such stray effects, since no conducting or insulating supports are required in the vicinity of the mouth, where the field is strong, and the coaxial transmission line enters the pipe at the closed end remote from the radiating opening. It is to be expected that the actual radiation pattern will be almost entirely that produced by the open end and that spurious radiation will be singularly absent. Under such conditions, a theoretical calculation, assuming the theory to be sound, should naturally agree closely with the practical results.

The smooth beam evidenced in Fig. 9 may be considered to be the result of the half-sinusoid distribution (1) across the mouth appropriate

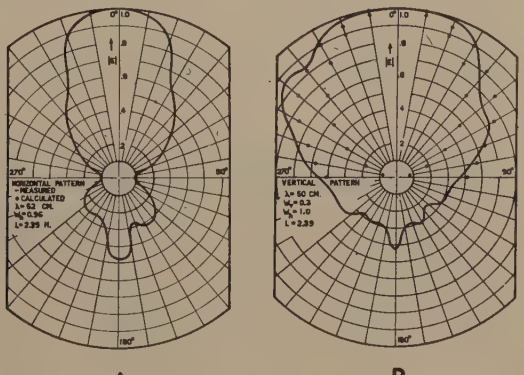

Fig. 10—Measured radiation patterns (solid curves) and calculated values (small circles). The solid dots are calculated by an incomplete method, as explained in the text.

to the $H_{0,1}$ wave. The pipe used in obtaining these patterns was 4.78 meters long and experiment had demonstrated that the distribution across the mouth was substantially the same as (1). It was also known that appreciable departures from this distribution obtained at the mouth of a pipe only half as long, and consequently one would expect a modified radiation pattern from such a pipe. That this occurs may be seen from Fig. 10A, which is the measured horizontal pattern at $\lambda = 52$ centimeters from a pipe of length 2.39 meters. The indentations in the beam may be attributed directly to the nonsinusoidal distribution across the mouth of the pipe. If smooth calculable patterns like those of Fig. 9 are to be obtained, it is clear that the pipe must be at least long enough for the desired type of wave to be established near the open end. In some instances, nevertheless, the shorter pipe may be preferred.

Unfortunately, the only measured vertical pattern was taken with a short pipe, L = 2.39 meters, and it displays the same kind of irregularities just discussed in connection with Fig. 10A. This vertical pattern, shown in Fig. 10B, is for the same cross section as the other diagrams, for which $W_v = 0.3$. It is relatively nondirective in the forward 180-degree sector. The small circles represent values calculated from (4). These values do not agree very satisfactorily with those of the measurements. This discrepancy is thought to be caused by radiation from conduction currents mentioned above, on the outer surfaces of the top and bottom induced by diffraction around the top and bottom edges. With larger apertures, W_v, a better agreement of theory with experiment is to be expected. It is an unexplained fact that the values of the y component of electric intensity E_y in the radiation field calculated directly from (1) by means of Huygens' principle, plotted as solid dots in Fig. 10B, agree very well with the measured values of E'_θ. This agreement, which has not been theoretically justified, is believed to be purely fortuitous.

Other Hollow-Pipe Radiators

The simple arrangement of Fig. 1. is but one of the many types of open-pipe radiators. As another example, a bidirectional radiator may be obtained that sends beams in each direction along the axis by locating the exciting rod near the center of a section of hollow pipe open at *both* ends.

A vertically polarized beam may be radiated from a rectangular pipe with a vertical $H_{0,1}$ wave. Similarly, a horizontally polarized beam may be radiated with a horizontal $H_{0,1}$ wave, that is, with a $H_{1,0}$ wave. These two waves are mutually independent in the pipe, and consequently two noninterfering beams can be radiated from the same pipe by providing it with a multiplex terminal device, similar to that illustrated[7] where the exciting rod 1 provides a horizontally polarized beam and the exciting rod 2 a vertically polarized beam. It is also possible with such a radiating system, by connecting the two exciting rods through their feed lines to appropriate apparatus, to radiate an elliptically or a circularly polarized beam.

Arrays of open hollow pipes, in which several pipes are disposed in space in some specific way and in which the exciting rod or rods are supplied with currents of specific amplitudes and phases, allow the production of radiation patterns of more complicated configurations. In this application, the individual open hollow pipes may be considered as the elements of the array, much as half-wave antennas constitute

[7] See Fig. 19 of footnote 2.

the elements for conventional short-wave arrays. Fig. 11A illustrates an array in which four hollow pipes have their terminal devices connected to the apparatus by coaxial lines. It is also possible to connect the hollow pipes directly to a single terminal device in the manner indicated in Fig. 11B, where a Y branch joins the two open pipes to the terminal device. In this form, the vertical pipe section can be made strong mechanically and can thus serve as the support or mast for the radiator as well as for the conductor of the electromagnetic energy.

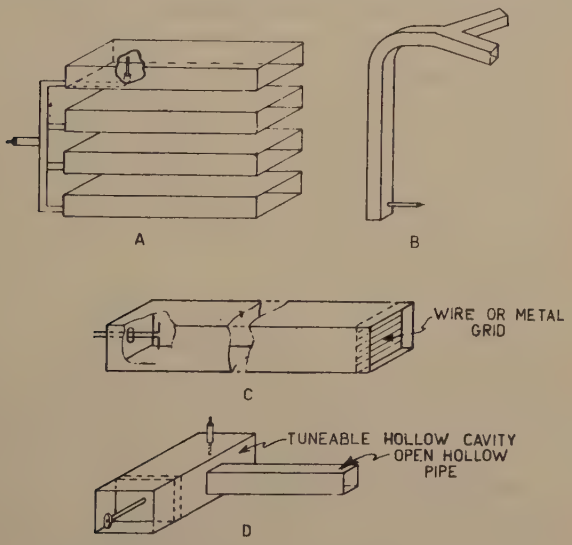

Fig. 11—Other forms of hollow-pipe radiators.

Another illustration of a hollow-pipe radiator is shown in Fig. 11B, where a grid of parallel wires or a slotted sheet-metal grill is located in the pipe or at the mouth. By passing waves of one polarization, vertical in the figure, and blocking waves of other polarizations, this grid increases the polarization selectivity of the radiator. By disposing the wires in the vertical direction, the resonant properties of the hollow-pipe cavity contained between the walls, the plunger, and the grid may be highly accentuated. Such a grid might be placed between the exciting rods[8] to insure further the independence of the vertically and the horizontally polarized waves. Another form of highly resonant termination by means of which the radiator can be made responsive to a very narrow range of frequencies to the exclusion of frequencies outside this range is illustrated in Fig. 11D. In this figure, a tunable hollow-pipe cavity or resonator with a plunger and an exciting rod is

[8] See Fig. 19 of footnote 2.

connected to the open hollow-pipe radiator. There is an analogy between this combination and a conventional open-wire antenna with a resonant circuit connected in its down-leads.

Conclusion

Two principal results of this research are the following: (1) the development of a theory for the radiation from open hollow pipes; and (2) the experimental verification of this theory. Another result is the justification of the application of Huygens' principle when used in conjunction with the Hertzian vector to radiation problems where the dimensions of the system are comparable to the wavelength. We believe also to have shown that the open rectangular pipe is a radiating element for ultra-short waves possessing several advantages over conventional types of antennas. The absence of insulating members, the freedom from stray radiation from supports and transmission lines, and the absence of difficult amplitude and phase adjustments are examples of these features.

Appendix

Derivation of the Expressions for the Radiation Pattern

The essential details of the analysis leading to (4) and (5) are here presented. This analysis is limited to the $H_{0,m}$ wave. In accordance with the summary of the method given in the body of the paper, we start with the Hertzian vector Π for the space inside the pipe, which has only one component in the y direction. Using the symbol (i) to represent $\sqrt{-1}$ and i, j, k to denote unit vectors in the x, y, z directions, respectively, the Hertzian vector within the pipe may be obtained in the following form:

$$\Pi = jC_1 \left\{ \begin{array}{c} \cos\left(\frac{m\pi}{b} z\right) \\ \\ \sin\left(\frac{m\pi}{b} z\right) \end{array} \right\} e^{-(i)\beta x + (i)\omega t}. \tag{10}$$

This value is assumed to exist also at the open end of the pipe. The same convention as to m will be used as explained under (1). The electric and magnetic field intensities are derived from the Hertzian vector by means of the vector relations

$$\begin{aligned} H &= \epsilon \operatorname{curl} \dot{\Pi} \\ E &= -\frac{1}{c^2} \ddot{\Pi} + \operatorname{grad} \operatorname{div} \Pi \end{aligned} \tag{11}$$

where ϵ = dielectric constant, c = velocity of light in vacuum, and the dot represents differentiation with respect to time. For example, (1) is obtained by substituting (10) into (11) and carrying out the indicated operations.

Huygens' principle in the formulation given by Kirchhoff is now used to obtain the Hertzian vector, Π' for the free space a great distance away from the pipe mouth. A suitable reference for an elementary exposition of Huygens' principle is "Introduction to Theoretical Physics," by Slater and Frank, Chapters XXVI and XXVII. In the form used in this reference, Huygens' principle is

$$\Pi' = \frac{1}{4\pi}\int\int \frac{1}{r}\left\{\left[\frac{1}{c}\left(\frac{\partial \Pi}{\partial t}\right)_{t-r/c} + \frac{\Pi\left(t-\frac{r}{c}\right)}{r}\right]\cos(p, r) + \left(\frac{\partial \Pi}{\partial p}\right)_{t-r/c}\right\} dS. \tag{12}$$

The normal to the plane of the mouth into the pipe, i.e., in the $(-x)$ direction, is denoted by p. The vector, r, connects the element of surface dS in the plane of the mouth with the observation point P. The retarded time is denoted by $t-r/c$. The integral is to be taken over a closed surface about the radiator. It will be taken only over the mouth with the distribution (10). It should be taken over the entire outer surface of the pipe also, but, unfortunately, we have no means of determining the value of Π on this outer surface. This defect is impossible to remove and is characteristic of the application of Huygens' principle to diffraction problems.

Evaluating the terms required in (12) from the value of Π from (10) gives

$$\Pi' = \frac{C_1(i)j}{4\pi}\int\int \frac{1}{r} e^{(i)\omega(t-r/c)} \left\{\begin{matrix}\cos\left(\frac{m\pi}{b}z\right) \\ \sin\left(\frac{m\pi}{b}z\right)\end{matrix}\right\} \cdot\left[\frac{\omega}{c}\cos(p, r) + \beta\right] dS. \tag{13}$$

The integral is next simplified by introducing the polar co-ordinates (see Fig. 5)

$$\begin{aligned} x_0 &= \bar{r}\cos\theta \\ y_0 &= \bar{r}\sin\theta\cos\zeta \\ z_0 &= \bar{r}\sin\theta\sin\zeta \end{aligned} \tag{14}$$

and approximating r by the expression

$$\begin{aligned} r &= \sqrt{x_0^2 + (y_0 - y)^2 + (z_0 - z)^2} \\ &= \sqrt{\bar{r}^2 - 2\bar{r}\sin\theta(y\cos\zeta + z\sin\zeta) + y^2 + z^2} \\ &\cong \bar{r} - \sin\theta(y\cos\zeta + z\sin\zeta). \end{aligned} \tag{15}$$

We assume that waves from all points of the mouth of the pipe arrive at the point P with the same amplitude and, hence we take $1/r$ outside of the integral. Finally, we treat cos (p,r) as a slowly varying function compared to the exponential and take it, too, outside the integral, writing it as cos θ. We drop the factor (i) outside the integral, since the absolute magnitude only is desired in the end. With these simplifications, we get

$$\Pi' = \frac{jC_1\omega}{4\pi\bar{r}c}\left(\cos\theta + \frac{c\beta}{\omega}\right)e^{(i)\omega(t-\bar{r}/c)} \cdot \int_{-b/2}^{b/2}\int_{-a/2}^{a/2}\left\{\begin{matrix}\cos\left(\frac{m\pi}{b}z\right) \\ \sin\left(\frac{m\pi}{b}z\right)\end{matrix}\right\} e^{(i)(2\pi/\lambda)(y\cos\zeta + z\sin\zeta)\sin\theta}\,dydz. \tag{16}$$

After integration we obtain

$$\Pi' = \frac{jC_1\omega}{4\pi\bar{r}c}\left(\cos\theta + \frac{\beta c}{\omega}\right)\frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\cos\zeta\right)}{\frac{\pi a}{\lambda}\sin\theta\cos\zeta} \cdot \left[\frac{\sin\left(\frac{\pi b}{\lambda}\sin\theta\sin\zeta + \frac{m\pi}{2}\right)}{\frac{\pi b}{\lambda}\sin\theta\sin\zeta + \frac{m\pi}{2}} + (-1)^{m+1}\frac{\sin\left(\frac{\pi b}{\lambda}\sin\theta\sin\zeta - \frac{m\pi}{2}\right)}{\frac{\pi b}{\lambda}\sin\theta\sin\zeta - \frac{m\pi}{2}}\right]e^{(i)\omega(t-\bar{r}/c)}, \tag{17}$$

m odd or even integer.

This is the general expression for the Hertzian vector of the radiation field from the open hollow pipe. After some manipulation and rearrangement, it can be put in the simplified form

$$\Pi' = \frac{jC_1\omega}{4\pi\bar{r}c}(\cos\theta + M)\left[\frac{\sin(A\cos\zeta)}{A\cos\zeta}\right]\cdot\left[\frac{\sin\left(B\sin\zeta + \frac{m\pi}{2}\right)}{(B\sin\zeta)^2 - \left(\frac{m\pi}{2}\right)^2}\right]e^{(i)\omega(t-\bar{r}/c)}. \tag{18}$$

There is only one component, the j component in the y direction, to Π'. We denote the complex amplitude of this component by Π_y', i.e., $\Pi' = j\Pi_y'$, and resolve the Hertzian vector for the radiation field into spherical co-ordinates, as follows:

$$\left.\begin{aligned} \Pi' &= i_{\bar{r}}\Pi_{\bar{r}}' + i_\theta\Pi_\theta' + i_\zeta\Pi_\zeta' \\ \Pi_{\bar{r}}' &= \Pi_y'\cos\zeta\sin\theta \\ \Pi_\theta' &= \Pi_y'\cos\zeta\cos\theta \\ \Pi_\zeta' &= -\Pi_y'\sin\zeta \end{aligned}\right\} \tag{19}$$

where $i_{\bar{r}}$, i_θ, i_ζ designate unit vectors in the $\bar{r}$, θ, and ζ directions. From (11), in spherical co-ordinate form, we obtain the electric and magnetic intensities in the radiation field by neglecting terms proportional to powers of $1/\bar{r}$ higher than the first. In this way, we get

$$\left.\begin{aligned} H &= -\frac{i_\theta}{\bar{r}\sin\theta}\frac{\partial}{\partial\bar{r}}(\bar{r}\sin\theta\,\Pi_\zeta') + \frac{i_\zeta}{\bar{r}}\frac{\partial}{\partial\bar{r}}(\bar{r}\Pi_\theta') \\ &= -(i)\frac{\omega^2\epsilon}{c}\{i_\theta\Pi_\zeta' - i_\zeta\Pi_\theta'\} \\ H_\theta &= -\frac{\omega^2\epsilon}{c}\Pi_\zeta' = \frac{\omega^2\epsilon}{c}\Pi_y'\sin\zeta \\ H_\zeta &= \frac{\omega^2\epsilon}{c}\Pi_\theta' = \frac{\omega^2\epsilon}{c}\Pi_y'\cos\zeta\sin\theta \end{aligned}\right\} \tag{20}$$

$$\left.\begin{aligned} E &= \left(\frac{\omega}{c}\right)^2\Pi' + i_{\bar{r}}\frac{\partial}{\partial\bar{r}}\left[\frac{1}{\bar{r}^2\sin\theta}\frac{\partial}{\partial\bar{r}}(\bar{r}^2\sin\theta\,\Pi_{\bar{r}}')\right] \\ &= \left(\frac{\omega}{c}\right)^2\Pi' - i_{\bar{r}}\left(\frac{\omega}{c}\right)^2\Pi_{\bar{r}}' = \left(\frac{\omega}{c}\right)^2[i_\theta\Pi_\theta' + i_\zeta\Pi_\zeta'] \\ E_\theta &= \left(\frac{\omega}{c}\right)^2\Pi_\theta' = \left(\frac{\omega}{c}\right)^2\Pi_y'\cos\zeta\cos\theta \\ E_\zeta &= \left(\frac{\omega}{c}\right)^2\Pi_\zeta' = -\left(\frac{\omega}{c}\right)^2\Pi_y'\sin\zeta \end{aligned}\right\}. \tag{21}$$

We observe that no radial components of intensity appear and that the wave is transverse, which is an essential characteristic of a radiation field in free space. We note, too, that the characteristic wave impedance is the same as that of a plane wave in free space:[4]

$$z' = \frac{E_\theta}{H_\zeta} = -\frac{E_\zeta}{H_\theta} = \frac{1}{c\epsilon} = c\mu = \sqrt{\frac{\mu}{\epsilon}} = 377 \text{ ohms}. \quad (22)$$

The resultant intensities in space, E' and H', are

$$\begin{aligned} E' &= \sqrt{E_\theta^2 + E_\zeta^2} \\ H' &= \sqrt{H_\theta^2 + H_\zeta^2}. \end{aligned} \quad (23)$$

For the purposes of our work with hollow-pipe radiators, however, the horizontal and the vertical characteristics are desired, which are given by the expressions

Horizontal characteristic (x, z *plane*; $\zeta = \pi/2$),

$$\begin{aligned} E_\theta &= 0, & H_\theta &= \frac{\omega^2\epsilon}{c}\Pi_y' \\ E_\zeta &= \left(\frac{\omega}{c}\right)^2 \Pi_y', & H_\zeta &= 0. \end{aligned} \quad (24)$$

Vertical characteristic (x, y *plane*; $\zeta = 0$),

$$\begin{aligned} E_\theta &= \left(\frac{\omega}{c}\right)^2 \Pi_y' \cos\theta, & H_\theta &= 0 \\ E_\zeta &= 0, & H_\zeta &= \frac{\omega^2\epsilon}{c}\Pi_y' \cos\theta. \end{aligned} \quad (25)$$

The absolute magnitudes of the functions E_ζ of (24) and E_θ of (25) are the quantities that were measured in the experiments. They have been written explicitly in (5) and (6).

Efforts have been made to calculate the radiation field in several other ways. One way was to assume a fictitious current sheet $I = \epsilon(\partial E_y/(\partial t)$ across the open mouth and to integrate the radiation from each elementary area of this sheet to obtain the resultant field at a great distance from the pipe. This method does not give the factor $(\cos\theta + M)$, but otherwise leads to the above derived expressions. Another way was to calculate the electric field at a great distance from the pipe by using E_y from (1) directly in Huygens' principle. This method gives the same values for the vertical and horizontal characteristics (4) and (5), but it does not give the correct polarization; i.e.,

the wave is not transverse. It appears to be almost impossible to complete this latter calculation, and hence this method is to be avoided in preference to the method presented above.[9]

This use of Huygens' principle *with the Hertzian vector*, which is experimentally confirmed by these experiments, is a severe and unusual test, because the velocity of phase propagation is not the same inside the pipe as it is in the same medium in the outer space, and also because the aperture may be comparable to or even smaller than the wavelength. Neither of these conditions is encountered in the usual applications of Huygens' principle in optics.

[9] An application of Huygens' principle to an ultra-short-wave radiation problem has recently been made by Diamond and Dunmore[10] in connection with the investigation of an underground antenna in a pit. It would appear that their solution, derived from the electric intensity alone, and not from the Hertzian vector or the vector and scalar potentials, should not give the correct polarization and pattern of the wave. An earlier application to parabolic reflectors was made by Darbord.[11]

[10] H. Diamond and F. W. Dunmore, "Experiments with underground ultra-high-frequency antenna for airplane landing beam," *Nat. Bur. Stand. Jour. Res.*, vol. 19, pp. 1–19; July, (1937). (Paper RP1006). Proc. I.R.E., vol. 25, pp. 1542–1560; December, (1937).

[11] R. Darbord, "Reflecteurs et lignes de transmission pour ondes ultra-courtes," *L'Onde Elec.*, vol. 11, pp. 53–82; February, (1932).

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ELECTROMAGNETIC WAVES IN HOLLOW METAL TUBES OF RECTANGULAR CROSS SECTION*

By

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Summary—*The theory of the transmission of electromagnetic waves in hollow conducting pipes of rectangular cross section is derived for perfectly conducting and for imperfectly conducting materials. Special attention is given to the type of wave that has only one transverse and everywhere parallel component of electric intensity, and the results of experiments on this wave are reported. Expressions for the attenuation of the different types of waves allow comparisons to be made with corresponding waves in a pipe of circular cross section. In a rectangular pipe, no wave has been found whose attenuation decreases indefinitely as the frequency is increased, as does the H_0 wave in a circular pipe.*

Introduction

RAYLEIGH'S pioneer paper[1] dealing with the propagation of electromagnetic waves through the inside of conducting tubes or pipes is now forty years old. It assumed the most idealized conditions of an infinitely long nonconducting cylinder of arbitrary dielectric constant embedded in a perfectly conducting material; no suggestion of a practical device was made, nor was the analysis capable of predicting the behavior of realizable tubes. In two later patents relating to transmission on a dielectric wire or rod, first proposed by Hondros and Debye,[2] a metal-sheathed dielectric[3] wire was described in which the dielectric material has a value substantially greater than that of air. It is only within the last year or so, however, following the discovery that the losses in a conducting tube may be tremendously reduced by making its interior hollow, i.e., of air, of gas, or evacuated, that it has received serious consideration as a practical device of probable importance in communications. At the present time there is an

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[1] Lord Rayleigh, "On the passage of electric waves through tubes, or the vibrations of dielectric cylinders," *Phil. Mag.*, vol. 43, pp. 125–132; February, (1897).

[2] D. Hondros and P. Debye, "Elektromagnetische Wellen an dielektrischen Drahten," *Ann. der Phys.*, vol. 32, pp. 465–476; June, (1910).

[3] British Patent No. 420,447; French Patent No. 757,972.

active interest in the hollow-pipe system, stimulated by the expectancy of its practical application to electrical communication. Following the papers by Southworth,[4] Carson, Mead and Schelkunoff,[5] and Barrow,[6] other papers[7,8,9,10,11,12] have discussed further details of the problem. The issuance of several patents[12] is an indication of possible commercial applications.

The hollow pipe of rectangular cross section, with which this paper deals, was discussed theoretically by Rayleigh[1] and by Brillouin[7] for an idealized conductor of infinite conductivity, and recently by Schelkunoff,[13] to some extent, for actual finitely conducting pipes. In this paper, the complete theory of transmission is derived without restriction, in a manner intended to clarify its physical significance. In addition, certain experiments are reported and some of the practical aspects of the problem are discussed. A comparison is made of rectangular and circular cross-section pipes as to the attenuation, operating frequencies, and application on the basis of equal cost.

In the interest of clarity, the paper has been divided into two parts, the first dealing with the idealized case of conductors of infinite conductivity and the second with the actual case of conductors of finite conductivity. Special emphasis has been placed on a type of wave, here termed the $H_{0,1}$ wave, that has the electric field intensity directed parallel to the y axis and at right angles to the direction of propagation. The mathematical analysis has been given in detail for this case only, but the final results are presented for waves of other types and these results are interpreted and compared with similar results for a circular pipe.

[4] G. C. Southworth, "Hyper-frequency wave guides—general considerations and experimental results," *Bell Sys. Tech. Jour.*, vol. 15, pp. 284–309; April, (1936).

[5] J. R. Carson, S. P. Mead, and S. A. Schelkunoff, "Hyper-frequency wave guides—mathematical theory," *Bell Sys. Tech. Jour.*, vol. 15, pp. 310–333; April, (1936).

[6] W. L. Barrow, "Transmission of electromagnetic waves in hollow tubes of metal," Proc. I.R.E., vol. 24, pp. 1298–1328; October, (1936).

[7] Léon Brillouin, "Propagation d'ondes électromagnetiques dans un tuyan," *Rev. Gen. de Élec.*, vol. 40, pp. 227–239; August, (1936).

[8] G. C. Southworth, "Some new forms of electrical transmission," *Eng. Jour.* (Canada), vol. 20, pp. 186–190; April, (1937).

[9] G. C. Southworth, "Some fundamental experiments with wave guides," Proc. I.R.E., vol. 25, pp. 807–822; July, (1937).

[10] L. Page and N. I. Adams, Jr., "Electromagnetic waves in conducting tubes," *Phys. Rev.*, vol. 52, pp. 647–651; September, (1937).

[11] G. C. Southworth, "New experimental methods applicable to ultra-short waves," *Jour. App. Phys.*, vol. 8, pp. 660–665; October, (1937).

[12] British Patents: 462,804; 463,238; 464,185; 466,063; U. S. Patent, 2,088,749.

[13] S. A. Schelkunoff, "Transmission Theory of plane electromagnetic waves," Proc. I.R.E., vol. 25, pp. 1457–1492; November, (1937).

PART I

Perfectly Conducting Rectangular Pipes

Fig. 1 represents a perspective sectional view of a hollow rectangular pipe or tube assumed to be so long that all end effects are negligible and to have a wall thickness sufficiently great to prevent currents from reaching the outside surface of the pipe. The inner dimensions are $y=a$, $z=b$ on the Cartesian co-ordinates x, y, z. The interior space

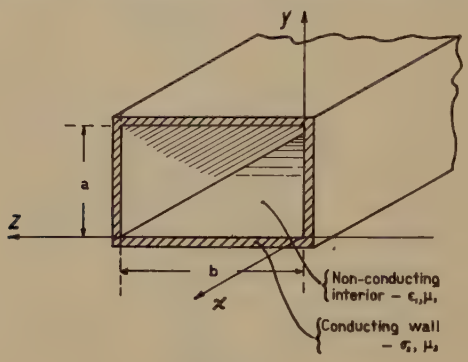

Fig. 1.

is characterized electrically by the dielectric constant ϵ_1 and permeability μ_1 and the pipe wall by the conductivity σ_2 and permeability μ_2. In this section of the paper, σ_2 will be assumed infinite, i.e., the pipe will be assumed to be a perfect conductor. Based on practical applications, for which losses must be kept at a minimum value, a pipe filled with air, a gas, or evacuated will receive principle attention; in this case $\epsilon_1=\epsilon_0$, where ϵ_0 applies to free space. A practical system of units will be used in which

$\mathfrak{E}$ = electric field intensity in volts per centimeter.

$\mathfrak{H}$ = magnetic field intensity in amperes per square centimeter.

σ = conductivity in mhos per centimeter.

μ = permeability in henrys per centimeter (for air, $\mu=\mu_0=4\pi\times10^{-9}$).

ϵ = dielectric constant in farads per centimeter (for air, $\epsilon=\epsilon_0=10^{-11}/36\pi$).

$c=1/\sqrt{\mu_0\epsilon_0}$ = velocity of light in free space in centimeters per second $=3\times10^{10}$.

The quantities $\mathfrak{E}$ and $\mathfrak{H}$ are real. For convenience in analysis, the complex field intensities E and H will be employed throughout, but it should be kept in mind that, in the end, the real part of E and H must be taken. Furthermore, it is convenient to introduce the factor $e^{-hx+i\omega t}$

into the field quantities to provide a wave traveling in the $+x$ direction, and to denote the complex amplitudes by E and H we have

$$\mathfrak{E} = \text{real } [\underline{E}] = \text{real } [\underline{E}e^{-hx+i\omega t}]$$
$$\mathfrak{H} = \text{real } [\underline{H}] = \text{real } [\underline{H}e^{-hx+i\omega t}].$$

In these expressions, ω and t have their usual significance, $i=\sqrt{-1}$, and $h=\alpha+i\beta$ denotes the propagation constant in which α is the attenuation constant and β is the phase constant. There will generally be components of $\mathfrak{E}$ and $\mathfrak{H}$ in the x, y, and z directions.

Previous papers have brought out the possible independent existence in perfectly conducting hollow pipes of two types of solutions that differ by the presence or absence of an x component of either the electric or the magnetic field intensity. In terms of their field components, these two solutions are characterized as follows:

$$E \text{ wave} \begin{cases} H_x = 0, \quad E_x \neq 0 \\ E_y, E_z, H_y, H_z \cdots \text{nonzero} \end{cases} \tag{1}$$

$$H \text{ wave} \begin{cases} E_x = 0, \quad H_x \neq 0 \\ E_y \text{ and } H_z, E_z \text{ and } H_y, \text{ or } E_y, E_z, H_y, H_z \cdots \text{nonzero.} \end{cases} \tag{2}$$

Solutions having the character of (1) may be interpreted as waves that have a longitudinal component of electric field but no such component of magnetic field, and solutions according to (2) lead to waves that have a longitudinal component of magnetic field but no such component of electric field. Rayleigh originally termed the phenomena associated with (1) "oscillations of the first kind" and those associated with (2) "oscillations of the second kind," but later workers have changed this nomenclature, using definitions based on the configuration of the electromagnetic field of the wave. At present, there are several different and rather confusing designations in use. For example, the following names and definitions, it is understood, are all intended to describe the same type of wave:

Electric Wave (E_0, E_1, $\cdots$); "—because there is a component of electric force in the direction of propagation"; Southworth, *Bell Sys. Tech. Jour.*, vol. 15, p. 286, (1936).

E-wave (E_0, E_1, $\cdots$); "—in which the axial component of magnetic force is everywhere absent—"; Carson, Mead, and Schelkunoff, *Bell. Sys. Tech. Jour.*, vol. 15, p. 313, (1936).

Longitudinal Wave; "—the field is describable by E_z, E_r, and H_ϕ"; Barrow, Proc. I.R.E., vol. 24, p. 1302, (1936).

Transverse Magnetic Wave; "—if the magnetic vector is perpendicular to the ray—" ["A ray is a straight line or curve normal to the equiphase surfaces;—"]; Schelkunoff, Proc. I.R.E., vol. 25, p. 1458, (1937).

Obviously, careful definition and usage by workers in this field is neces-

sary if confusion is to be avoided, as was recently pointed out,[13] and therefore we shall state clearly what terms we will use and what they will mean.

While stating definitions for the hollow pipe, it is helpful to raise several general considerations. Electromagnetic waves may be classified in several ways. One way is to classify them broadly according to their field characteristics and without reference to the medium (whether homogeneous, isotropic, etc.), to the disposition of the boundaries (whether a wave in unlimited space, a wave guided along a half space, guided along wires, etc.) or to the nature of the boundaries (whether conducting, nonconducting, etc.). Such a broad classification is that introduced by Schelkunoff,[13] and it is one that is admirably suited to the purposes of mathematical analysis. The engineering side of wave transmission is, however, much concerned with the material aspects of the medium and of the disposition and nature of the boundaries of the system, and from this standpoint a classification that contemplates the shape and materials of the system along which the waves are guided is also desirable. Consequently, we shall first form a classification according to these characteristics and then define the different types of waves in the hollow-pipe system according to the configuration of their field patterns.

Physically realizable electromagnetic waves may be divided into two major classes, viz, *free waves* and *guided waves*. The radiation from a dipole in unlimited space is an example of the former class and waves on a Lecher system illustrate the latter. The energy of free waves spreads more or less in all directions in space, but that of a guided wave is confined to the immediate vicinity of the guiding system. The strength of free waves varies with the distance r as an inverse power of r, for example, $1/r$, but that of a guided wave varies as $e^{-\alpha r}$. The rate of variation of the guided wave depends on α and may be made arbitrarily small by suitably choosing the materials of the guiding system. In this paper we are concerned with guided waves.

All waves that may be propagated within any hollow conducting pipe or tube will be called *hollow-pipe waves*. By this designation, we differentiate the electromagnetic phenomena associated with hollow-pipe transmission from those effecting transmission over a solid conducting cylinder or wire, which are well known as *wire waves*, and over other wave-guiding systems. Thus, we may also speak of *coaxial-conductor waves and parallel-wire waves*. It is probably because only one type of wave occurs in these transmission systems in normal operation that such a designation has not been generally used. Transmission phenomena associated with a nonconducting dielectric cylinder have

long been referred to as *dielectric wire waves*,[2] and they are not the same as hollow-pipe waves, since they have different field configurations and obey different laws. This striking difference appears to be partly obscured in several recent papers in which hollow conducting tubes and solid dielectric cylinders are treated as if they were identical systems and termed "wave guides." Furthermore, there appears to be no need for this restricted use of the term "wave guide," because a single wire, a parallel line, a concentric line, and even the earth's surface, are just as much wave guides.

Any hollow-pipe wave having both a longitudinal and a transverse component of electric field but only a transverse component of magnetic field will be called an *E wave*. Waves having components in accordance with (1) are thus *E* waves.

Any hollow-pipe wave having both a longitudinal and a transverse component of magnetic field but only a transverse component of electric field will be called an *H wave*. Waves in accordance with (2) are *H* waves. These symbols were first used in this connection by Carson, Mead, and Schelkunoff.[5]

These definitions are based on the idealized case of perfectly conducting pipes. It is convenient to apply them also to realizable metal pipes, although strictly neither wave can exist alone in a finitely conducting pipe. A further notation to supply information as to the shape of the pipe cross section is not thought desirable. However, we shall use subscripts to denote the harmonic order of the wave in each coordinate, as will be explained later. Generally, there will be two subscripts, although in special cases more may be necessary. In rectangular pipes, we need one for each side, hence we shall refer to $E_{n,m}$ waves and $H_{n,m}$ waves.

$E_{n,m}$ Waves

We desire the expressions for the field intensities, the critical frequencies, the velocities of propagation and other pertinent quantities for the $E_{n,m}$ waves in perfectly conducting pipes of rectangular cross section. The method of analysis may be found in Rayleigh[1] and subsequent publications and will not be repeated here. The field intensities inside the pipe are given by the expressions

$$E_x = A \sin\left(\frac{n\pi}{a}y\right) \sin\left(\frac{m\pi}{b}z\right) e^{-hx+i\omega t}$$

$$E_y = -A\frac{h}{k_1^2+h^2}\left(\frac{n\pi}{a}\right) \cos\left(\frac{n\pi}{a}y\right) \sin\left(\frac{m\pi}{b}z\right) e^{-hx+i\omega t}$$

$$E_z = -A\frac{h}{k_1^2+h^2}\left(\frac{m\pi}{b}\right)\sin\left(\frac{n\pi}{a}y\right)\cos\left(\frac{m\pi}{b}z\right)e^{-hx+i\omega t} \quad (4)$$

$$H_y = A\frac{i\omega\epsilon_1}{k_1^2+h^2}\left(\frac{m\pi}{b}\right)\sin\left(\frac{n\pi}{a}y\right)\cos\left(\frac{m\pi}{b}z\right)e^{-hx+i\omega t}$$

$$H_z = -A\frac{i\omega\epsilon_1}{k_1^2+h^2}\left(\frac{n\pi}{a}\right)\cos\left(\frac{n\pi}{a}y\right)\sin\left(\frac{m\pi}{b}z\right)e^{-hx+i\omega t}$$

$$H_x = 0.$$

A is a constant denoting absolute magnitude and depends on the excitation only. If $n=0$, $m=0$, all six components vanish, as also occurs if $n=0$, $m=1$ or $n=1$, $m=0$; thus, there are no possible waves of the types $E_{0,0}$, $E_{0,1}$, or $E_{1,0}$. With $n=1$, $m=1$, we obtain physically realizable waves of the $E_{1,1}$ type. They have the lowest critical frequency of all E waves in rectangular pipes, but the lowest-order H wave has a still lower value. Waves of $E_{1,2}$, $E_{2,1}$ and all higher-order types are theoretically possible. Waves having complementary indexes, like 1, 2

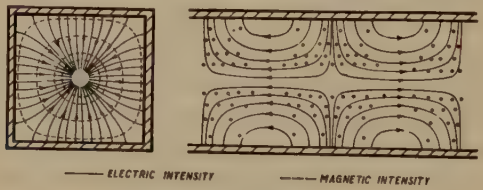

Fig. 2—$E_{1,1}$ wave.

and 2, 1 are alike except for their orientation in the pipe, and it is sufficient to consider one of them. The index n simply indicates the number of half sinusoids or maxima of the field intensity distribution that is found along the y axis from 0 to a, and m indicates the similar quantity along the z axis between 0 and b.

The configuration of the field in the $E_{1,1}$ wave may be found by a procedure similar to that used in an earlier paper,[14] although the solution in the present case can be obtained in analytic form. The lines of electric and of magnetic intensity are shown in Fig. 2 for a square pipe ($a=b$). In a rectangular pipe ($a\neq b$), the lines of force are bent from the location shown so as to terminate orthogonally on the pipe wall, but they retain the same general appearance. The $E_{1,1}$-wave configuration is the unit from which all other E waves may be constructed by a simple process like building a block house. For example, the $E_{1,2}$ wave has a field structure resulting from two $E_{1,1}$ waves being adja-

[14] Page 1310 of reference 6.

cently placed in one pipe, as illustrated in Fig. 3. Higher-order waves are formed in an obvious manner. All of the lines of electric intensity in an $E_{1,1}$ wave terminate on the conducting wall of the pipe; however, some of these lines in the $E_{1,2}$ wave do not do so, being closed on themselves.

When the operating frequency is above the critical value, the propagation constant is imaginary for perfectly conducting pipes,

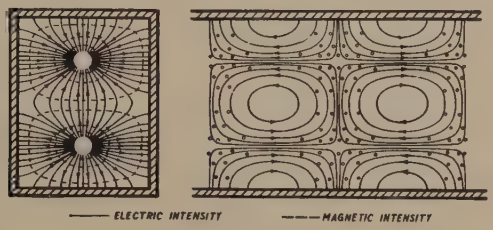

Fig. 3—$E_{1,2}$ wave.

$h=i\beta$, where β is the phase constant, and it may be obtained by an application of the boundary conditions, giving

$$k_1^2 + h^2 = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2. \tag{5}$$

From the value of h from (5), one obtains the following quantities for transmission by E waves in rectangular hollow pipes:

$$\begin{aligned}
&\text{phase constant} && \beta = \sqrt{\omega^2\mu_1\epsilon_1 - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2} \\
&\text{critical frequency} && f_0 = \frac{1}{2\sqrt{\mu_1\epsilon_1}}\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2} \\
&\text{critical wavelength} && \lambda_0 = 2/\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2} \\
&\text{wavelength in pipe} && \lambda = \frac{2\pi}{\beta} \\
&\text{phase velocity} && v_p = \frac{\omega}{\beta} \\
&\text{group velocity} && v_g = \frac{d\omega}{d\beta} = \frac{\beta}{\omega\mu_1\epsilon_1} = \frac{v_1^2}{v_p}.
\end{aligned} \tag{6}$$

We note that the product of the phase and group velocities equals the square of the velocity v_1 of a free wave in a medium with the same

constants as the inside of the pipe; or, if air is inside, $v_g \cdot v_p = c^2$. The critical frequency is lowest for the lowest-order wave and it increases as either n or m is increased; i.e., as the order of the wave becomes larger. It also increases as either dimension, a or b, is decreased. It may be decreased by using a dielectric material inside the pipe that has a large dielectric constant ϵ_1, but from the standpoint of obtaining low attenuation, only air, a gas, or a vacuum appears to have practical significance at the present time. Table I gives the first several critical frequencies and wavelengths for the special case of an air-filled square pipe for both E and H waves.

TABLE I

CRITICAL VALUES FOR AN AIR-FILLED SQUARE PIPE ($a=b$)

Wave type	Critical frequency f_c in cycles	Critical wavelength λ_c in centimeters
$H_{0,1}$	$1.50\times10^{10}/b$	$2.000\times b$
$H_{1,1}$ and $E_{1,1}$	$2.12\times10^{10}/b$	$1.413\times b$
$H_{0,2}$	$3.00\times10^{10}/b$	$1.000\times b$
$H_{1,2}$ and $E_{1,2}$	$3.35\times10^{10}/b$	$0.896\times b$
$H_{2,2}$ and $E_{2,2}$	$4.24\times10^{10}/b$	$0.718\times b$
$H_{0,3}$	$4.50\times10^{10}/b$	$0.666\times b$
$H_{1,3}$ and $E_{1,3}$	$4.74\times10^{10}/b$	$0.632\times b$
$H_{2,3}$ and $E_{2,3}$	$5.41\times10^{10}/b$	$0.554\times b$
$H_{3,3}$	$6.00\times10^{10}/b$	$0.500\times b$

$H_{n,m}$ Waves

For the H waves according to (2), the field intensities inside a perfectly conducting pipe are found to be

$$
\begin{aligned}
H_x &= B \cos\left(\frac{n\pi}{a}y\right)\cos\left(\frac{m\pi}{b}z\right)e^{-hx+i\omega t} \\
H_y &= B\frac{h}{k_1^2+h^2}\left(\frac{n\pi}{a}\right)\sin\left(\frac{n\pi}{a}y\right)\cos\left(\frac{m\pi}{b}z\right)e^{-hx+i\omega t} \\
H_z &= B\frac{h}{k_1^2+h^2}\left(\frac{m\pi}{b}\right)\cos\left(\frac{n\pi}{a}y\right)\sin\left(\frac{m\pi}{b}z\right)e^{-hx+i\omega t} \qquad (7) \\
E_y &= B\frac{i\omega\mu_1}{k_1^2+h^2}\left(\frac{m\pi}{b}\right)\cos\left(\frac{n\pi}{a}y\right)\sin\left(\frac{m\pi}{b}z\right)e^{-hx+i\omega t} \\
E_z &= -B\frac{i\omega\mu_1}{k_1^2+h^2}\left(\frac{n\pi}{a}\right)\sin\left(\frac{n\pi}{a}y\right)\cos\left(\frac{m\pi}{b}z\right)e^{-hx+i\omega t} \\
E_x &= 0.
\end{aligned}
$$

B is a constant depending on the strength of excitation. If $n=0$, $m=0$, no wave is possible because all six components vanish. When $n=0$, $m\neq0$, one finds that $H_y=E_z=E_x=0$ and the number of field components reduces to three, consequently all waves of the $H_{0,m}$ type

are possible. The $H_{0,m}$ waves are characterized by the fact that the electric field intensity is transverse to the direction of propagation and is everywhere parallel to the y axis, because there is only one electric component, E_y. Several other features make this type of wave out-

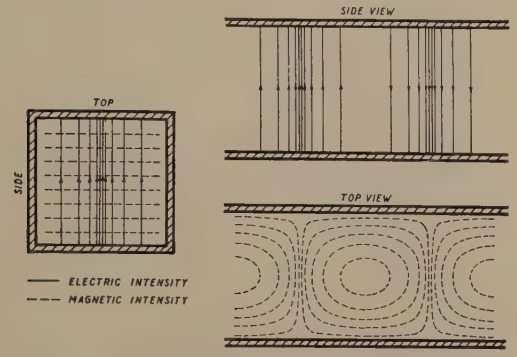

Fig. 4—$H_{0,1}$ wave.

standing in practical applications, as will be discussed later. Waves of the type $H_{1,1}$ and waves of all higher indexes are theoretically possible, but the higher-order waves imply such enormous frequencies that their practical application, even their realization, is doubtful.

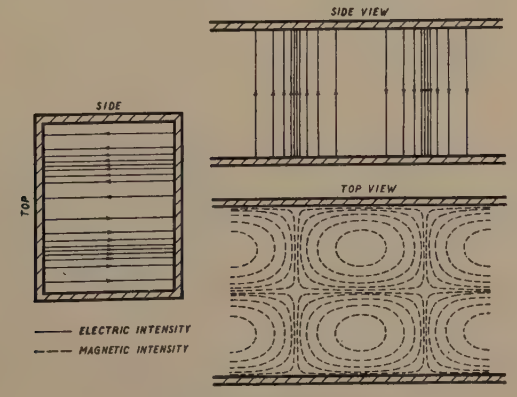

Fig. 5—$H_{0,2}$ wave.

The field configuration of the H waves may be found in the customary manner. Fig. 4 shows the unusually simple pattern of the $H_{0,1}$ wave in a square pipe. The transverse electric field has a sinusoidal intensity distribution over the cross section in the z direction. It can be excited by a length of straight current-carrying conductor placed trans-

verse to the x axis and parallel to the pipe side, provided that the frequency exceeds f_0. Fig. 5 shows the configuration of the $H_{0,2}$ wave in a rectangular pipe. These figures show how the $H_{0,m}$ waves, $m>1$, are built up of units similar to the $H_{0,1}$ wave. Fig. 6 shows the $H_{1,1}$ wave,

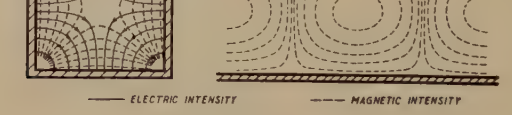

Fig. 6—$H_{1,1}$ wave.

which serves as a building unit for the $H_{1,2}$ and all higher-order H waves, as may be appreciated by a study of the pattern of the $H_{1,2}$ wave shown in Fig. 7.

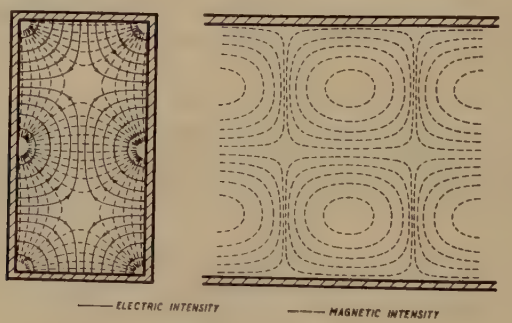

Fig. 7—$H_{1,2}$ wave.

A determination of the expressions for β, f_0, λ_0, λ, v_p, and v_g for the H waves results in the expressions already given in (6) for the E waves. Thus, the transmission quantities are identically the same for E and H waves of the same order. The critical values f_0 and λ_0 for the lower-order H waves in a square pipe are also given in Table I.

Terminal Devices for Rectangular Pipes

Terminal devices for exciting or for receiving any of the hollow-pipe waves in rectangular pipes can be constructed in the manner of those illustrated in Figs. 8 to 13; the terminal devices in the figures correspond to the waves illustrated in Figs. 2 to 7. The basic principle in the construction is that a conducting rod, or rods, carrying current of the proper frequency is disposed inside the pipe so as to coincide with a line of electric intensity of the field pattern for the desired wave, consideration being given to the phase of the currents when

two or more rods are employed. Obviously, exciting rods for $H_{n,m}$ waves *must* be at right angles to the direction of propagation and those for the $E_{n,m}$ waves are preferably positioned along this direction. Oppositely directed arrows next to the rods indicate a relative phase angle of the respective currents of 180 degrees. In these terminals, this phase angle is provided by using appropriate lengths of coaxial line to feed the rods, but other means could be used. The length of rods and their distance from the conducting wall that closes the end of the pipe are

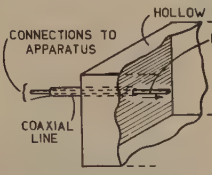

Fig. 8—$E_{1,1}$ wave.

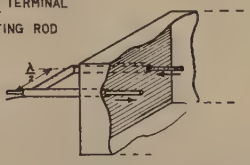

Fig. 9—$E_{1,2}$ wave.

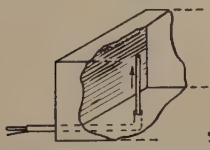

Fig. 10—$H_{0,1}$ wave.

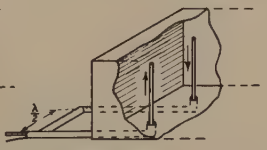

Fig. 11—$H_{0,2}$ wave.

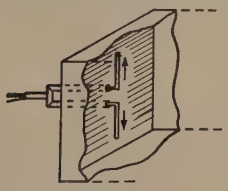

Fig. 12—$H_{1,1}$ wave.

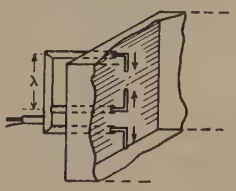

Fig. 13—$H_{1,2}$ wave.

also important design quantities and should be properly adjusted in each case; in fact, provision for adjustment is desirable, which may be accomplished by plungers or by movable exciting rods. Parallel-wire feed can be used when desired, or the vacuum tube, crystal detector, or other device can be connected as a part of the rod and placed inside the tube.

$H_{0,1}$ *Waves*

As pointed out in the introduction, this type of wave possesses such outstanding characteristics that it deserves further discussion. First, it has the simplest configuration of all hollow-pipe waves, possessing only the one transverse component of electric intensity E_y; second, it has the lowest critical frequency; and third, it has the smallest attenuation, as will be shown later.

The expressions for the $H_{0,1}$ wave in an air-filled pipe may be written as follows:

$$
\begin{aligned}
E_y &= Bi\omega\mu_1\left(\frac{b}{\pi}\right)\sin\left(\frac{\pi}{b}z\right)e^{-i\beta x+i\omega t} \\
H_x &= B\cos\left(\frac{\pi}{b}z\right)e^{-i\beta x+i\omega t} \\
H_z &= Bi\beta\left(\frac{b}{\pi}\right)\sin\left(\frac{\pi}{b}z\right)e^{-i\beta x+i\omega t} \\
H_y &= E_x = E_y = 0 \\
\beta &= \sqrt{\left(\frac{\omega}{c}\right)^2-\left(\frac{\pi}{b}\right)^2} \\
f_0 &= c/2b \\
\lambda_0 &= 2b \\
\lambda &= \lambda_0/\sqrt{1-\left(\frac{\lambda_e}{2b}\right)^2} \\
v_p &= c/\sqrt{1-\left(\frac{\lambda_e}{2b}\right)^2} \\
v_g &= c\sqrt{1-\left(\frac{\lambda_e}{2b}\right)^2}
\end{aligned}
\tag{8}
$$

where λ_e is the wavelength of the excitation in free space.

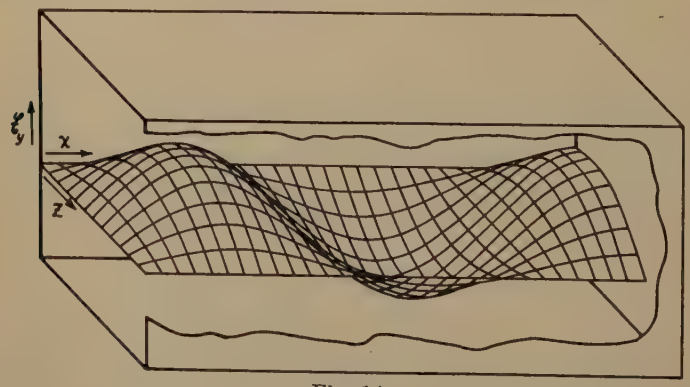

Fig. 14

The real electric field intensity $\mathfrak{E}_y$, and also the real displacement current, is independent of y, but it varies sinusoidally in the x and z directions. If we represent the value of $\mathfrak{E}_y$ by the vertical distance

from a plane through the pipe, we get the surface shown in Fig. 14, which further helps to visualize the $H_{0,1}$ wave. In most of the other waves, the field configuration is too complicated for such a representation.

When an open-ended rectangular pipe, or a flared rectangular horn on the end of such a pipe, is used to radiate electromagnetic waves, it may be assumed that this simple $H_{0,1}$ wave configuration produces a displacement current sheet across the mouth of the pipe or horn that has the current elements everywhere parallel to themselves. Since the polarization of the component space waves radiated by each element is the same, radiation will take place with maximum effectiveness. No other type of hollow-pipe wave possesses this feature.

Fig. 15

The critical wavelength is given by the simple expression $\lambda_0 = 2b$. This result is striking, because λ_0 depends only on the dimension of *one* side of the pipe, namely, the dimension in the z direction at right angles to the electric vector, and consequently the dimension of the other or a side may be made arbitrarily small without altering the critical values. For a given operating frequency, the rectangular pipe with the $H_{0,1}$ wave may have a much smaller cross-sectional area and contain less material than would be required with a circular or a square pipe using any type of wave.

A complete experimental verification[15] of the field pattern, the critical wavelength, and the velocity of phase propagation of the $H_{0,1}$ wave has been made. Fig. 15 shows an experimental rectangular pipe with sides $a = 15$ centimeters, $b = 50$ centimeters. One section 2.44 meters long is shown, but other similar sections can be connected together to make a longer pipe. The dimension b can be changed by moving the sides horizontally. The oscillator, visible at the right end

[15] The authors wish to acknowledge their indebtedness to Mr. J. D. Parker and to Mr. A. E. Whitcomb for their assistance in carrying out these experiments as part of their thesis work at The Massachusetts Institute of Technology.

of the photograph, was developed especially for this purpose. It comprises a Western Electric 316-A triode in a circuit with coaxial tuning elements and it covers continuously the range from 40 to 125 centimeters. It was modulated at 60 cycles and connected to the exciting rod inside the pipe by a coaxial line. The electric field intensity was measured by a small probe with a crystal detector, followed by an audio-frequency amplifier and a copper-oxide meter.

Other experiments have been made at wavelengths of about 12.5 centimeters with ordinary 2×3 inch rectangular galvanized-iron gutter downspouting, which is cheap and readily obtainable, as the hollow conductor. That there is no difficulty in transmitting waves around bends has been demonstrated by using the regular 90-degree elbows in a hollow-pipe system of this kind. It was also interesting to observe that transmission was not materially disturbed by separating the ends of a joint by several centimeters.

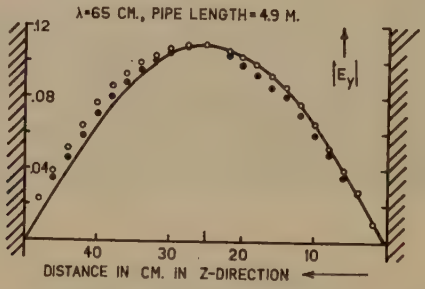

Fig. 16—$H_{0,1}$ wave.
——— sine curve. o o o in plane of the mouth.
• • • 45 centimeters back from mouth.

Fig. 16 shows typical measurements of the distribution of electric intensity $\mathfrak{E}_y$ across the inside of the pipe. In this case, the pipe was about 7.5 wavelengths long and measurements were made at various distances back from the mouth and across the mouth. An approximately sinusoidal distribution was found, indicating that the distortion of the wave, even at the mouth, was very small. However, when the pipe was about 3.7 wavelengths long, appreciable deviations from the sinusoidal distribution were observed that were greater in the vicinity of the exciting rod than near the open end. It is clear that the pipe must be at least several wavelengths long before the hollow-pipe waves achieve their theoretical form. Fig. 17 shows measured values and the theoretical curve for the wavelength in the pipe as a function of the wavelength λ_e of the excitation in free space. The wavelength in the pipe is greater than λ_e, hence it follows that the phase velocity in

the air-filled pipe is always greater than that of light, but, as seen in the figure, the phase velocity approaches light velocity asymptotically as λ_e becomes very small.

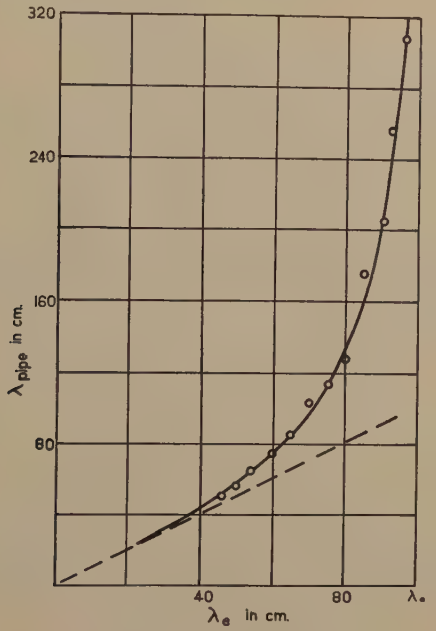

Fig. 17—$H_{0,1}$ wave.

The longitudinal current along the upper or lower half of the pipe is found to be

$$I = \int_o^b H_z\, dz = 2Bi\beta\left(\frac{b}{\pi}\right)^2 \tag{9}$$

and the voltage between the top and bottom sides in the y direction at the center of the pipe is

$$V = \int_0^a \underline{E_y}\left(\frac{b}{2}\right)dy = iB\omega\mu_1 ab/\pi. \tag{10}$$

From the complex Poynting vector, we calculate the average power flowing through the pipe

$$S_T = \int \tfrac{1}{2}[E_y \times H_z^*]dA = \tfrac{1}{4}\,|\,B\,|^2\omega\mu_1\beta\left(\frac{b}{\pi}\right)^2 ab, \tag{11}$$

where the asterisk denotes the conjugate quantity. If $|E_y|$ and $|H_z|$ are held constant, the transmitted power increases directly with the

cross-sectional area of the pipe. A characteristic impedance may be defined in any of several ways, and each expression differs somewhat from the others. *On a power basis*,[5] we get

$$Z_0 = \frac{S_T}{I_{rms}^2} = \frac{\omega\mu_1\pi^2 a}{8\beta b} \tag{12}$$

but *on a current-voltage basis*

$$Z_0 = \frac{V}{I} = \frac{\omega\mu_1\pi a}{2\beta b} . \tag{13}$$

We may also define a quantity something like an impedance that is characteristic of the particular wave type *on a field intensity basis*

$$Z_0' = \frac{E_y}{H_z} = \frac{\omega\mu_1}{\beta} = \frac{\mu_1 c}{M} \tag{14}$$

where

$$M = \sqrt{1 - \left(\frac{\lambda_c}{2b}\right)^2} .$$

For purposes of comparison, we note that the similar quantity for a plane wave in free space is

$$Z' = \frac{E}{H} = \mu_1 c = 377 \text{ ohms} \tag{15}$$

and that Z_0' approaches this value as the dimension b of the pipe is made very large compared to the wavelength of the excitation λ_c.

$H_{0,1}$ Wave: Resolution into Elementary Waves

It is convenient for several purposes to change the expressions for the $H_{0,1}$ wave from their form (8) to an equivalent form in which this wave appears as a superposition of two sets of ordinary plane waves that are multiply reflected back and forth between the side walls of the pipe. This resolution was first described by Brillouin,[7] to whom the authors are indebted for the idea, and it has also been employed by Page and Adams.[10] The other types of waves may be resolved in a similar manner, but we shall not reproduce such calculations in this paper. This new form lends itself readily to physical picturization; however, its particular importance here is that it provides a relatively clear and straightforward way of calculating the attenuation in the pipe.

We start from the field expressions (8), but we rewrite them to correspond to an x axis through the center of the pipe, as shown in Fig. 18,

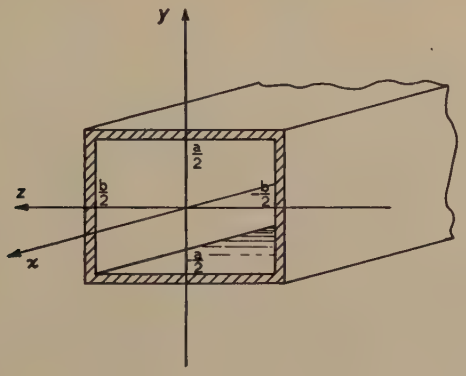

Fig. 18

$$
\begin{aligned}
E_y &= iB\omega\mu_1 \left(\frac{b}{\pi}\right) \cos\left(\frac{\pi}{b}z\right) e^{-i\beta x+i\omega t} \\
H_x &= -B \sin\left(\frac{\pi}{b}z\right) e^{-i\beta x+i\omega t} \qquad (16) \\
H_z &= iB\beta \left(\frac{b}{\pi}\right) \cos\left(\frac{\pi}{b}z\right) e^{-i\beta x+i\omega t}.
\end{aligned}
$$

Let us consider for a moment the E_y component alone and, by using the exponential form of the cosine, write it as

$$
\begin{aligned}
E_y &= iB\omega\mu_1 \left(\frac{b}{\pi}\right) \frac{1}{2} \left[e^{i[(\pi/b)z-\beta x]+i\omega t} + e^{-i[(\pi/b)z+\beta x]+i\omega t}\right] \\
&= iB\omega\mu_1 \left(\frac{b}{\pi}\right) \frac{1}{2} [\quad I \quad + \quad II \quad].
\end{aligned} \qquad (17)
$$

In (17), the two exponential terms I and II may be interpreted as two component waves traveling in separate directions. The direction of the first component wave I may be found from $((\pi/b)\,z-\beta x)$ and the geometric representation of Fig. 19 (a) to make the angle θ with the x axis, where

$$
\theta = \tan^{-1}\frac{\pi}{b\beta} = \tan^{-1}\frac{1}{\sqrt{\left(\frac{f}{f_0}\right)^2 - 1}}. \qquad (18)
$$

We next rotate the z, x axes through this angle θ to the new directions z', x', as illustrated in Fig. 19(b), so that x' will correspond to the direction of propagation of the first component wave. This rotation is ac-

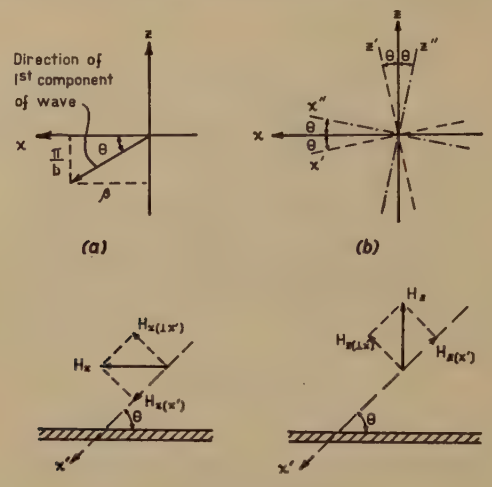

Fig. 19

complished by substituting for z and x the values

$$\begin{aligned} z &= z' \cos\theta - x' \sin\theta \\ x &= x' \cos\theta + z' \sin\theta, \end{aligned} \tag{19}$$

whereupon we get for the component I

$$I = e^{i[(\pi\cos\theta/b-\beta\sin\theta)z'-(\pi\sin\theta/b+\beta\cos\theta)x']+i\omega t}.$$

Putting into this expression the values for $\cos\theta$ and $\sin\theta$ from Fig. 19(a) and the value of β from (8) leads to the expression

$$I = e^{-i(\omega/c)x'+i\omega t}. \tag{20}$$

If we carry out a similar operation on the second component wave II, and denote the new direction by x'', which is rotated in the opposite sense from x by the same angle θ, as indicated in Fig. 19(b) ,we obtain an expression like (20) for the second component, viz,

$$II = e^{-i(\omega/c)x''+i\omega t}. \tag{21}$$

Equations (20) and (21) substituted in (17) give E_y. A similar resolution may also be carried out for H_x and H_z. In this way we obtain the following modified expressions for the $H_{0,1}$ wave:

$$E_y = \left[iB\omega\mu\left(\frac{b}{\pi}\right)\frac{1}{2}\right] e^{i\omega(-(x'/c)+t)} + \left[iB\omega\mu\left(\frac{b}{\pi}\right)\frac{1}{2}\right] e^{i\omega(-(x''/c)+t)}$$

$$H_x = [iB\tfrac{1}{2}]e^{i\omega(-x'/c+t)} - [iB\tfrac{1}{2}]e^{i\omega[-(x'/c)+t]} \qquad (22)$$

$$H_z = \underbrace{\left[iB\beta\left(\frac{b}{\pi}\right)\frac{1}{2}\right] e^{i\omega(-(x'/c)+t)}}_{\text{first group}} + \underbrace{\left[iB\beta\left(\frac{b}{\pi}\right)\frac{1}{2}\right] e^{i\omega[-(x''/c)+t]}}_{\text{second group}}.$$

In this form, two groups of terms appear; the first group represents a wave traveling in the x' direction, or from right to left when facing in the x direction, and the second group represents a wave in the x'' direction, or from left to right. E_y is transverse to both directions. We shall show that in each group the magnetic field is also transverse. Take the first group: Resolve H_x into two components, one in the x' direction and the other in a direction at right angles thereto, denoted by $H_{x(x')}$ and $H_{x(\perp x')}$ respectively; treat H_z in the same way; add the like directed components (refer to Fig. 19(c)):

$$H_{x(x')} + H_{z(x')} = 0$$

$$H_{x(\perp x')} + H_{z(\perp x')} = H_{z'} = \left[iB\left(\frac{b}{\pi}\right)\frac{\omega}{c}\frac{1}{2}\right] e^{i\omega(-(x'/c)+t)}. \qquad (23)$$

Clearly, the magnetic field intensity is perpendicular to the x' direction, i.e., it is directed along the z' axis and we shall denote it by $H_{z'}$, as in (23). Thus, from the first group, we obtain the final expressions

$$H_{0,1} \text{ elementary wave right to left} \begin{cases} E_y = \left[iB\omega\mu_1\left(\frac{b}{\pi}\right)\frac{1}{2}\right] e^{i\omega(-(x'/c)+t)} \\ H_{z'} = \left[iB\frac{\omega}{c}\left(\frac{b}{\pi}\right)\frac{1}{2}\right] e^{i\omega(-(x'/c)+t)} \end{cases} \qquad (23a)$$

and in an analogous manner from the second group, we get

$$H_{0,1} \text{ elementary wave left to right} \begin{cases} E_y = \left[iB\omega\mu_1\left(\frac{b}{\pi}\right)\frac{1}{2}\right] e^{i\omega(-(x''/c)+t)} \\ H_{z''} = \left[iB\frac{\omega}{c}\left(\frac{b}{\pi}\right)\frac{1}{2}\right] e^{i\omega(-(x''/c)+t)}. \end{cases} \qquad (23b)$$

The expressions (23a) and (23b) are completely equivalent to (8). Together, they describe an $H_{0,1}$ wave in an air-filled perfectly conducting pipe, but in this form the $H_{0,1}$ wave appears as a superposition of two crisscross *plane waves*, which we shall term *elementary waves*, traveling

at the normal velocity of light c and being reflected back and forth between the two walls $z=\pm b/2$. The wave (23a), representing a wave traveling from right to left when facing the x direction, may be considered as striking the left-hand pipe wall at $z=-b/2$ at the angle of incidence θ and the wave (23b) may be considered as the wave reflected

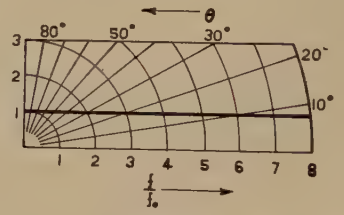

Fig. 20

from this surface at an angle θ and traveling to the right. At the right-hand wall, $z=+b/2$, the rôles of the two waves are interchanged. In subsequent paragraphs, we shall make use of this conception to calculate the transmission loss. The curve relating the angle of incidence θ and the ratio f/f_0 is shown in polar co-ordinates in Fig. 20 by the heavy line. At the critical frequency $f=f_0$, $\theta=90$ degrees, and the elementary plane waves bounce back and forth from wall to wall without advancing in the x direction. When f is just above f_0 in value, $\theta<90$

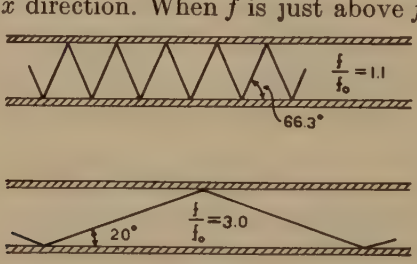

Fig. 21

degrees and there is a slow progression in the x direction, as indicated diagrammatically in Fig. 21. As the frequency f is increased, the angle θ becomes less, reaching 20 degrees when $f/f_0=3.0$, Fig. 21, etc. As pointed out by Brillouin[7] and by Page and Adams,[10] this behavior of the elementary waves makes beautifully clear the physical significance of the critical frequency, of the group velocity which is less than c, and of the phase velocity which is greater than c.

PART II

Imperfectly Conducting Pipes

Actually, with metal pipes, the conductivity is not infinite, as assumed heretofore for simplicity, but it has a finite value that depends

on the metal. There appears to be no chance of finding an exact solution when the conductivity is finite, because the boundary conditions cannot be formally satisfied on the four sides of the rectangular pipe. Nevertheless, we can obtain satisfactory approximate values for the attenuation for any value of conductivity and of wavelength of practical importance.

As a first approximation, we shall assume that the field inside the pipe is not appreciably distorted from its shape in a perfectly conducting pipe. This assumption is justified by the high conductivity of suitable metals and has been proved experimentally for rectangular pipes (Fig. 16, for example), and for circular pipes.[4,6] We are therefore able to start with the previously given expressions for the field for the case of perfect conductivity. We further simplify the problem by separately calculating the losses on the two sets of opposite walls, $\pm a$ and $\pm b$ respectively. While calculating the losses on either of the pairs of walls, the effect of the two remaining walls will be neglected. The problem is thus reduced to the study of waves between two parallel conducting planes. Between two parallel planes, two types of waves may exist independently; one type having the electric field and the other type having the magnetic field parallel to the planes and perpendicular to the direction of propagation. All such waves have a sinusoidal variation of field in the direction perpendicular to the planes.

All types of hollow-pipe waves in a rectangular pipe may be resolved into component waves each of which is one of the above types of parallel-plane waves. In the preceding section, we have given the resolution of the $H_{0,1}$ wave in detail. It was shown (23a,b) that this wave may be resolved into two component waves, each of which has the magnetic field parallel to the walls $y = \pm a/2$ and perpendicular to its new direction of propagation x' or x''; there was no harmonic space variation between the walls. Without further analysis, one may see that the electric field is parallel to the walls $z = \pm b/2$ and perpendicular to the original direction of propagation x; the space variation between the walls has the form $\cos((\pi/b)\, z)$.

The field expressions thus far derived must now be modified in order to satisfy the boundary conditions appropriate to finitely conducting walls. The modification can most conveniently and logically be made if we start with the elementary wave form (23a,b).

$H_{0,1}$ Wave: Calculation of Losses

The losses accompanying transmission by an $H_{0,1}$ wave in an imperfectly conducting pipe may be divided into the two following components:

Loss I. Losses on the top and bottom walls, $y = \pm a/2$.

Loss II. Losses on the two side walls, $z = \pm b/2$.

We shall calculate these losses separately. The medium inside the pipe is assumed to be a perfect nonconductor.

Loss I. Consider only the left-to-right elementary wave (23a) which has the two field components E_y and $H_{z'}$. The imperfect conductivity allows conduction current to flow into the walls, accompanied by a tipping forward of the electric vector and, therefore, by the presence of a second component of electric field $E_{x'}$. To modify (23a) for the case of imperfect conductivity, we use the solution of the wave equation between two parallel planes $y = \pm a/2$ for the magnetic field intensity, $H_{z'}$, and obtain the two components of electric field intensity, E_y and $E_{x'}$, from Maxwell's equations, giving

in the dielectric

$$H_{z'} = \left[\frac{i}{2} B \frac{\omega}{c} \frac{b}{\pi}\right] \cos (r_y y) e^{-h_{x'}x' + i\omega t} \tag{24a}$$

$$E_y = \frac{-h_{x'}}{i\omega\epsilon_1} H_{z'} = \frac{-h_{x'}}{i\omega\epsilon_1}\left[\frac{i}{2} B \frac{\omega}{c} \frac{b}{\pi}\right] \cos (r_y y) e^{-h_{x'}x' + i\omega t} \tag{24b}$$

$$E_{x'} = \frac{-1}{i\omega\epsilon_1} \frac{\partial H_{z'}}{\partial y} = \frac{r_y}{i\omega\epsilon_1}\left[\frac{i}{2} B \frac{\omega}{c} \frac{b}{\pi}\right] \sin (r_y y) e^{-h_{x'}x' + i\omega t} \tag{24c}$$

where $h_{x'}$ is the propagation constant in the x' direction. The complex quantities $h_{x'}$ and r_y satisfy the relation

$$\left(\frac{\omega}{c}\right)^2 + h_{x'}^2 - r_y^2 = 0,$$

obtained by substituting (24a) into the wave equation. As will be shown later, r_y is a very small complex number that approaches zero when the conductivity becomes infinite.

The velocity of propagation of the wave in the conductor in the x' direction must be the same as that of the wave in the dielectric, and therefore the constant $h_{x'}$ is the same for the metal as it is for the dielectric. In accordance with the assumption that the pipe wall is so thick that currents do not reach the outside surface, the wave in the metal will have an exponential propagation in the y direction also; we denote this propagation constant by h_y. The solution of the wave equation in the metal will thus have the form

$$H_{z'} = C_1 e^{-h_y y - h_{x'}x' + i\omega t} \tag{25a}$$

$$E_y = -\frac{h_{x'}}{\sigma_2} C_1 e^{-h_y y - h_{x'}x' + i\omega t} \qquad y \geqq a/2. \tag{25b}$$

$$E_{x'} = \frac{h_y}{\sigma_2} C_1 e^{-h_y y - h_{x'}x' + i\omega t} \tag{25c}$$

The constants $h_{x'}$ and h_y are related by the equation

$$-i\omega\mu_2\sigma_2 + h_{x'}^2 + h_y^2 = 0$$

obtained by substituting (25a, b, or c) into the wave equation for the metal. The quantity $h_{x'}$ has a value differing but little from its value $i(\omega/c)$ for perfect conductivity and hence it may be neglected compared to $i\omega\mu_2\sigma_2$, giving the approximate value for h_y for high conductivities

$$h_y \cong \sqrt{i\omega\mu_2\sigma_2} = \frac{1}{\sqrt{2}}(1+i)\sqrt{\omega\mu_2\sigma_2}. \tag{25d}$$

At the boundary $y=a/2$, the tangential components of E and of H must be equal:

$$\left.\begin{aligned} E_{x',\,\text{air}}(a/2) &= E_{x',\,\text{metal}}(a/2) \\ H_{z',\,\text{air}}(a/2) &= H_{z',\,\text{metal}}(a/2) \end{aligned}\right\}$$

and this condition gives the transcendental equations

$$\left.\begin{aligned} \frac{r_y}{i\omega\epsilon_1}\left[\frac{i}{2}B\frac{\omega}{c}\frac{b}{\pi}\right]\sin\left(r_y\frac{a}{2}\right) &= \frac{h_y}{\sigma_2}C_1 e^{-h_y(a/2)} \\ \left[\frac{i}{2}B\frac{\omega}{c}\frac{b}{\pi}\right]\cos\left(r_y\frac{a}{2}\right) &= C_1 e^{-h(a/2)} \end{aligned}\right\}. \tag{26a}$$

The value of r_y can be determined by taking the quotient of the two equations and using the first term of the series expansion for the tangent, as follows:

$$\begin{aligned} \tan\left(r_y\frac{a}{2}\right) &= \frac{h_y}{r_y}\frac{i\omega\epsilon_1}{\sigma_2} \\ r_y &= e^{i(3\pi/8)}\sqrt{\frac{2\omega\epsilon_1}{a}}\sqrt{\frac{\omega\mu_2}{\sigma_2}}. \end{aligned} \tag{27}$$

It follows that

$$h_{x'} = i\left(\frac{\omega}{c} + \frac{\omega\epsilon_1}{a}\sqrt{\frac{\omega\mu_2}{2\sigma_2}}\right) + \frac{\omega\epsilon_1}{a}\sqrt{\frac{\omega\mu_2}{2\sigma_2}}.$$

Squaring (26a) and (26b) and using the relation $\cos^2+\sin^2=1$ leads to the following value for C_1 for large values of σ_2:

$$C_1 = \frac{\left[\frac{i}{2}B\frac{\omega}{c}\frac{b}{\pi}\right]e^{h_y(a/2)}}{\left[1-\left(\frac{h_y\omega\epsilon_1}{r_y\sigma_2}\right)^2\right]^{1/2}} = \left[\frac{i}{2}B\frac{\omega}{c}\frac{b}{\pi}\right]e^{h_y(a/2)}. \tag{28}$$

The approximations made in arriving at (28) are justifiable for any case of σ_2 and ω of practical interest; for example, with a copper pipe, the expression is valid at $\lambda = 0.3$ centimeters. Substituting the value of C_1 from (28) into (25a, b, c), we get the desired expressions for the field in the metal

$$\text{in the conductor} \left\{ \begin{aligned} H_{z'} &= \left[\frac{i}{2} B \frac{\omega}{c} \frac{b}{\pi}\right] e^{-h_y(y-a/2)-h_{x'}x'+i\omega t} && \text{(29a)} \\ E_y &= -\frac{i\omega}{\sigma_2 c} \left[\frac{i}{2} B \frac{\omega}{c} \frac{b}{\pi}\right] e^{-h_y(y-a/2)-h_{x'}x'+i\omega t} && \text{(29b)} \\ E_{x'} &= \sqrt{\frac{i\omega\mu_2}{\sigma_2}} \left[\frac{i}{2} B \frac{\omega}{c} \frac{b}{\pi}\right] e^{-h_y(y-a/2)-h_{x'}x'+i\omega t} && \text{(29c)} \end{aligned} \right\} y \geqq \frac{a}{2}.$$

The energy flow into the metal per second per square centimeter of surface is

$$S = \tfrac{1}{2}[E_{x'} \times H_{z'}^*] = \tfrac{1}{2} |B|^2 \left(\frac{\omega b}{2\pi c}\right)^2 \sqrt{\frac{\omega\mu_2}{2\sigma_2}}.$$

The asterisk denotes the conjugate quantity. Since this energy loss is independent of the co-ordinates, we can get the total loss per centimeter length in the x direction by multiplying by b. The resultant value must be doubled to include the loss in the opposite wall. Now, the above loss was caused by the left-to-right elementary wave alone; an identical loss will result from the right-to-left elementary wave (24b), so that we must again double the above expression.[16] Thus, we have *the total loss S_I on the top and bottom walls of the pipe per centimeter length:*

$$S = 2b |B|^2 \left(\frac{\omega b}{2\pi c}\right)^2 \sqrt{\frac{\omega\mu_2^2}{2\sigma_2}}. \qquad (30)$$

Loss II. In calculating the loss on the two side walls $z = \pm b/2$, we return to the unresolved form (16) of the $H_{0,1}$ wave and note that the field components are propagated in the x direction and vary sinusoidally in the z direction, but are independent of the co-ordinate y. The polarization is such that the electric field is parallel to the walls $z = \pm b/2$ and perpendicular to the direction of propagation x. The waves travel

[16] It might at first appear that the sum of the losses from the two elementary waves is not equal to the total loss, since generally the sum of the losses computed from component fields is not equal to the loss computed from the resultant field. It has been proved, however, that this equality is correct for the average loss per centimeter of pipe length and hence that (30) really is the total value of Loss I.

along between the two side walls almost as if the top and bottom were not there. Neglecting any distortion caused by the top and bottom walls $y = \pm a/2$, the field components of the wave appear as follows:

$$\text{in the dielectric} \begin{cases} E_y = B \dfrac{i\omega\mu_1}{r_z} \cos(r_z z) e^{-h_x x + i\omega t} & (31a) \\ H_x = -B \sin(r_z z) e^{-h_x x + i\omega t} & (31b) \\ H_z = B \cos(r_z z) e^{-h_x x + i\omega t} & (31c) \end{cases} z \leqq b/2.$$

Both the propagation constant in the x direction h_x and the quantity r_z are complex numbers to be determined by the boundary conditions and are subject to the condition

$$\left(\frac{\omega}{c}\right)^2 + h_x^2 - r_z^2 = 0.$$

The wave penetrates into the metal in the z direction in addition to its motion in the x direction. Denoting the propagation constant in the z direction by h_z, the following solutions of Maxwell's equations must represent the field in the conductor bounded by $z = +b/2$:

$$\begin{cases} E_y = C_2 e^{-h_z z - h_x x + i\omega t} & (32a) \\ H_x = \dfrac{-h_z}{i\omega\mu_2} C_2 e^{-h_z z - h_x x + i\omega t} & (32b) \\ H_z = \dfrac{h_x}{i\omega\mu_2} C_2 e^{-h_z z - h_x x + i\omega t} & (32c) \end{cases} z \geqq b/2.$$

A similar set of equations with h_z replaced by $-h_z$ would represent the field in the other side wall. The propagation constants h_z and h_x are related by the equation

$$-i\omega\mu_2\sigma_2 + h_x^2 + h_z^2 = 0$$

or

$$h_z \cong \sqrt{i\omega\mu_2\sigma_2} \qquad (33)$$

since h_x is approximately equal to $i\sqrt{(\omega/c)^2 - (\pi/b)^2}$ and is negligible in this equation.

At the boundary $z = +b/2$, we must have

$$E_{y,\,\text{air}}(b/2) = E_{y,\,\text{metal}}(b/2)$$

$$H_{x,\,\text{air}}(b/2) = H_{x,\,\text{metal}}(b/2)$$

which gives the transcendental equations

$$\left.\begin{aligned} B\frac{i\omega\mu_1}{r_z}\cos\left(r_z\frac{b}{2}\right) &= C_2 e^{-h_z(b/2)} \qquad (34a)\\ B\sin\left(r_z\frac{b}{2}\right) &= \frac{h_z}{i\omega\mu_2}C_2 e^{-h_z(b/2)} \qquad (34b)\end{aligned}\right\}.$$

Dividing (34a) by (34b) and using the first term of the expansion for the cotangent and the approximate value of h_z gives

$$\cot\left(r_z\frac{b}{2}\right) = \frac{\mu_2 r_z}{\mu_1 h_z}$$

$$r_z = \frac{\pi}{b}\left(1 - \frac{b}{\mu_1}\sqrt{\frac{2\mu_2}{\omega\sigma_2}} + i\frac{b}{\mu_1}\sqrt{\frac{2\mu_2}{\omega\sigma_2}}\right)$$

$$h_x = i\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{b}\right)^2\left(1 - \frac{b}{\mu_1}\sqrt{\frac{2\mu_2}{\omega\sigma_2}}\right) + \frac{\pi^2}{2b\mu_1}\sqrt{\frac{2\mu_2}{\omega\sigma_2}}\left[\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{b}\right)^2\right]^{-1/2}}. \qquad (35)$$

When $\sigma_2 = \infty$, this expression will yield $r_z = \pi/b$, the value for the non-dissipative case. Solving (34a) and (34b) for C_2 by eliminating the trigonometric functions gives

$$C_2 = \frac{Be^{h_z(b/2)}}{\sqrt{\left(\frac{r_z}{i\omega\mu_1}\right)^2 + \left(\frac{h_z}{i\omega\mu_2}\right)^2}} \cong B\frac{i\omega\mu_2}{h_z}e^{h_z(b/2)} \qquad (36)$$

because $|h_z| \gg |r_z|$. Substituting (26) and (33) into (32) gives the expressions for the field in the conductor

in the conductor

$$\left.\begin{aligned} E_y &= B\sqrt{\frac{i\omega\mu_2}{\sigma_2}}\,e^{-h_z(z-b/2)-h_x x+i\omega t} \qquad (37a)\\ H_x &= -B\,e^{-h_z(z-b/2)-h_x x+i\omega t} \qquad (37b)\\ H_z &= B\frac{h_x}{\sqrt{i\omega\mu_2\sigma_2}}\,e^{-h(z-b/2)-h_x x+i\omega t} \qquad (37c)\end{aligned}\right\} z \geqq b/2.$$

Neglecting the real part of h_x, the energy flow into the metal per second per square centimeter on the surface $z = +b/2$ is

$$S = \tfrac{1}{2}[E_y \times H_x^*] = \tfrac{1}{2}|B|^2\sqrt{\frac{\omega\mu_2}{2\sigma_2}}.$$

This value is to be doubled and multiplied by a to get the total loss per centimeter length in the x direction. Thus, we have for the total loss S_{II} on the two side walls of the pipe per centimeter length:

$$S_{II} = a \mid B \mid^2 \sqrt{\frac{\omega\mu_2}{2\sigma_2}}. \tag{38}$$

$H_{0,1}$ Wave: Attenuation

Using the values from (11), (30), and (38), the attenuation constant α may now be calculated as follows:

$$\begin{aligned}\alpha &= \frac{1}{2} \cdot \frac{\text{power loss per centimeter of length}}{\text{power transmitted through interior}} \\ &= \frac{1}{2} \cdot \frac{S_I + S_{II}}{S_T} \\ &= K \cdot b^{-3/2} \left[\frac{\frac{1}{2}\frac{b}{a}\left(\frac{f}{f_0}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2 - 1}} + \frac{\left(\frac{f}{f_0}\right)^{-1/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2 - 1}} \right]\end{aligned} \tag{39}$$

where $K = \sqrt{2\pi\mu_2/c\sigma_2\mu_1^2}$, in nepers per centimeter. The first radical depends only on the materials of which the pipe is constructed, the second factor depends on the absolute magnitude of the side b, and the bracketed factor depends only on the two ratios f/f_0 and b/a.

The attenuation is here expressed as the sum of two parts, namely, the two terms in the brackets times the common factor, the first part arising from the losses on the top and bottom walls and the second part from losses on the two side walls; we shall call these two parts α_I and α_{II}, respectively. Both components are infinite at the critical frequency f_0, but just above this frequency they become smaller very rapidly as the frequency is increased. The first component α_I has a minimum value at $f/f_0 = \sqrt{3} = 1.73$, but it rises again to increase as the square root of the frequency for large values of f/f_0. This part, α_I, coming from the top and bottom walls, is responsible for an attenuation that increases indefinitely with increasing frequency in any rectangular pipe. The second term, α_{II}, has no minimum, but it continues to decrease with increasing frequency, being proportional to $f^{-3/2}$ for large values of f/f_0. For values of f/f_0 greater than about $\sqrt{3}$, only the component α_I is of consequence; i.e., most of the losses then take place on the top and bottom walls $y = \pm a/2$. When operating a rectangular pipe in this range of frequencies, therefore, it is highly impor-

tant to *make the inner surface of its top and bottom walls of a material of very high conductivity.*

Perhaps the most reasonable comparison of rectangular pipes of various forms of cross section is one in which the shape is changed, keeping the periphery, and thus also the amount of metal used in the construction and the cost, at some constant value. In this way, it has been found that the optimum ratio of a/b, i.e., the value that gives the minimum attenuation, is 1.18. The series of curves of α versus f reproduced in Fig. 22 shows this fact, as well as making clear the dependency

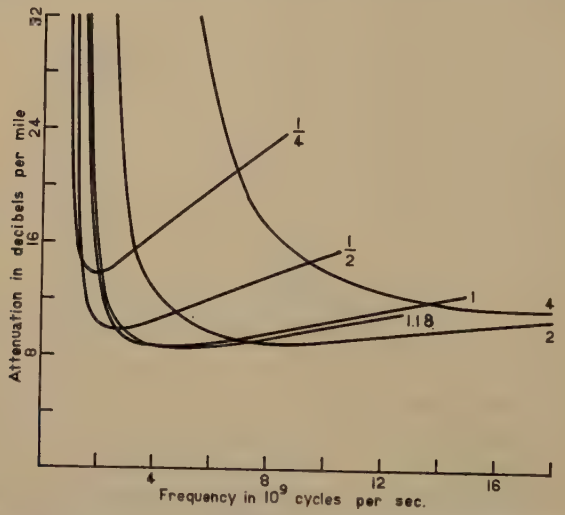

Fig. 22—$H_{0,1}$ wave in air-filled copper pipe for periphery of 40 centimeters for different a/b ratios.

of the attenuation on a/b and on the frequency; the curves are for an air-filled copper pipe having a periphery of 40 centimeters. It is noteworthy, however, that the variation of α with a/b in the region between about 1/2 and 2 is small, and consequently any ratio of a/b between these limits results in an attenuation of almost the minimum value. This finding indicates that small inaccuracies in the manufacture will be inconsequential; we may also infer that this conclusion will apply to circular pipes. For example, for a square pipe, $a/b=1$, the attenuation is only 1 per cent greater than the minimum value; however, the wavelength for minimum attenuation has increased by 17 per cent, compared to its value for $a/b=1.18$. The choice of pipe shape is also affected by the availability of suitable power sources at the high frequencies required, and perhaps a ratio of $a/b=1/2$ or even smaller, for which the attenuation is higher than $\alpha_{\min}$ but for which a much lower

operating frequency is possible, may prove desirable in certain cases. By flattening a square pipe of periphery $4b$ and critical wavelength $2b$ into the extreme shape, keeping its periphery constant, its critical wavelength approaches the limiting value of $4b$; i.e., it may be used at a wavelength twice as long as that required for the square shape. If the

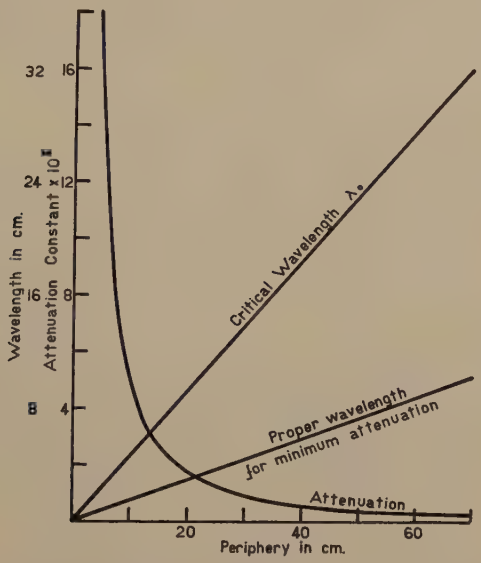

Fig. 23—$H_{0,1}$ wave in air-filled copper pipe of optimum a/b ratio = 1.18.

shape a/b is held constant, the attenuation varies as $b^{-3/2}$, according to (35). By equating the derivative of (35) to zero, we find that the value of f/f_0 for minimum attenuation is

$$\left(\frac{f}{f_0}\right)_{\text{opt}} = \sqrt{3\left(\frac{a}{b}+\frac{1}{2}\right)+\sqrt{9\left(\frac{a}{b}\right)^2+7\frac{a}{b}+\frac{9}{4}}}\,. \qquad (40)$$

For a square pipe, this ratio is 2.96, i.e., the minimum attenuation occurs at a frequency about three times the critical frequency, but the attenuation is almost the same over a tremendously wide frequency band. This band width, as well as the value of $(f/f_0)_{\text{opt}}$, increases with increasing a/b as seen in Fig. 22. Fig. 23 shows the variation with the periphery of the minimum attenuation and the corresponding values of optimum wavelength and critical wavelength for an air-filled copper pipe of the best a/b ratio (1.18).

Higher-Order Waves: Attenuation

Calcuation of the losses and the attenuation for waves of the $E_{n,m}$ and the $H_{n,m}$ types have been carried out along the same lines as those just explained, namely, by resolving the general n,m wave type into its equivalent elementary waves and treating the problem as though these elementary waves were guided between infinite parallel planes. The results of these calculations, which are rather involved, give the following expressions for the attenuation constant:

$H_{0,m}$ wave:

$$\alpha = K b^{-3/2}\sqrt{m}\left[\frac{\frac{b}{2a}\left(\frac{f}{f_0}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2-1}}+\frac{\left(\frac{f}{f_0}\right)^{-1/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2-1}}\right]$$

$H_{n,m}$ wave:

$$\alpha = K b^{-3/2}\sqrt{m}\,\frac{\frac{b}{a}\left\{1+\left(\frac{n}{m}\right)^2\frac{b}{a}\right\}\left(\frac{f}{f_0}\right)^{3/2}+\left\{1+\left(\frac{n}{m}\right)^2\left(\frac{b}{a}\right)^3\right\}\left(\frac{f}{f_0}\right)^{-1/2}}{\left[1+\left(\frac{b}{a}\right)^2\left(\frac{n}{m}\right)^2\right]^{3/4}\sqrt{\left(\frac{f}{f_0}\right)^2-1}} \qquad (41)$$

$E_{n,m}$ wave:

$$\alpha = K b^{-3/2}\sqrt{m}\,\frac{\left[1+\left(\frac{n}{m}\right)^2\left(\frac{b}{a}\right)^3\right]\left(\frac{f}{f_0}\right)^{3/2}}{\left[1+\left(\frac{b}{a}\right)^2\left(\frac{n}{m}\right)^2\right]^{3/4}\sqrt{\left(\frac{f}{f_0}\right)^2-1}}$$

A separate expression is required for the $H_{0,m}$ wave, because the one for the $H_{n,m}$ wave does not reduce to the correct value simply by substituting $n=0$. With this exception, these expressions agree exactly with those given by Schelkunoff.[13]

For a square pipe, $a=b$, with the same order of harmonic variation in each of the two dimensions of the cross section, the attenuation constants may be simplified as follows:

$$\text{square pipe, } n=m \begin{cases} H_{0,m} \text{ wave:} \\ \alpha = K b^{-3/2}\sqrt{m}\left[\dfrac{\frac{1}{2}\left(\frac{f}{f_0}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2-1}}+\dfrac{\left(\frac{f}{f_0}\right)^{-1/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2-1}}\right] \\ H_{m,m} \text{ wave:} \\ \alpha = K b^{-3/2}\sqrt{m}\sqrt[4]{2}\left[\dfrac{\left(\frac{f}{f_0}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2-1}}+\dfrac{\left(\frac{f}{f_0}\right)^{-1/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2-1}}\right] \\ E_{m,m} \text{ wave:} \\ \alpha = K b^{-3/2}\sqrt{m}\sqrt[4]{2}\,\dfrac{\left(\frac{f}{f_0}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2-1}} \end{cases} \quad (42)$$

In general, the attenuation constant is inversely proportional to the three-halves power of the linear dimensions of the tube and directly proportional to the square root of the order of the harmonic variation.

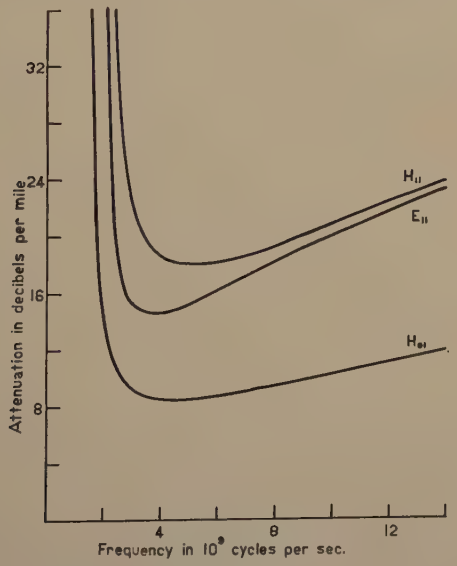

Fig. 24—For a square air-filled copper pipe $a=b=10$ centimeters.

For equal orders of harmonic variations, $n=m$, a square pipe gives the minimum attenuation for both H and E waves, which fact is

attributable to symmetry. In a square pipe, the $H_{0,1}$ wave has a smaller attenuation than does any other type of wave. In Fig. 24 are shown curves of attenuation constant versus frequency for the three lowest-order waves, $H_{0,1}$, $H_{1,1}$, and $E_{1,1}$ in a square air-filled copper pipe, 10 centimeters on a side. The critical and optimum conditions for these cases are given in Table II.

TABLE II

FOR A SQUARE AIR-FILLED COPPER PIPE, 10 CENTIMETERS ON A SIDE

Wave type	$H_{0,1}$	$H_{1,1}$	$E_{1,1}$
Critical frequency in cycles	1.50×10^9	2.12×10^9	2.12×10^9
Frequency for minimum attenuation in cycles	4.44×10^9	5.18×10^9	3.67×10^9
Minimum attenuation in decibels per mile	8.55	18.1	14.6

A study of the general expressions (41) discloses that there is no wave in a realizable rectangular pipe whose attenuation continues to decrease indefinitely with increasing frequency, as does that of the H_0 wave in a circular pipe as discovered by Carson, Mead, and Schelkunoff.[5] However, a study of (41) for rectangular pipes shows that if the ratio b/a approaches zero with b constant, that is, if the cross section is made very long in the y direction, the attenuation of $H_{0,m}$ and $H_{n,m}$ waves approaches the same exceptional form as that of the aforesaid H_0 wave and decreases indefinitely with increasing frequency. For this degenerate case, the attenuation expression becomes

$$\alpha = K b^{-3/2}\sqrt{m}\,\frac{(f/f_0)^{-1/2}}{\sqrt{\left(\frac{f}{f_0}\right)^2 - 1}}$$

and for high values of $f/f_0 > 1$ it has the form

$$\alpha = K b^{-3/2}\sqrt{m}\,\left(\frac{f}{f_0}\right)^{-3/2}.$$

Although this limiting case cannot be realized, it nevertheless has a practical implication in that the flat portion of the attenuation curve can be materially extended by employing a pipe with small b/a ratio, or large a/b ratio, as is evident from the $H_{0,1}$ wave from Fig. 23. The anomalous behavior of the attenuation in this particular case can be explained clearly with the aid of the conceptions already used in the calculation of the attenuation. The losses on top and bottom walls of the pipe are similar to those on a two-conductor transmission line, and are caused, at high frequencies, by an alternating-current resistance

that is proportional to $\sqrt{f}$, according to the usual skin-effect expression. By making the dimension a very large, the effect of top and bottom walls becomes very small and the wave acts like one multiply reflected between two parallel planes, viz., the side-wall conductors. Here, however, the incident angle decreases indefinitely with increasing frequency, resulting in a smaller number of reflections per unit length and in less absorption per reflection, because grazing incidence is approached. Consequently, in the limit, an anomalous attenuation function results.

Comparison of Square and Circular Pipes

Before giving quantitative results, we wish to discuss certain conclusions drawn by Brillouin.[7] In commenting on the H_0 wave in a *circular* pipe, he concludes that the exceptional inverse dependency of its attenuation on the frequency is the result of perfect symmetry and, therefore, that the linear superposition of an $H_{0,2}$ and an $H_{2,0}$ wave in a *square* pipe gives a resultant wave type corresponding to the H_0 wave and possessing a similar anomalous variation of attenuation with frequency. We have investigated this matter analytically and are forced to the opposite conclusion. Both the power loss, S_I+S_{II}, in the pipe walls and the power S_T transmitted through the pipe are doubled by the superposition of the two waves, hence the attenuation $\alpha=(S_I+S_{II})/2S_T$ remains unchanged at the value for the $H_{0,2}$ wave. We do not find any possible mode of transmission in a square or a rectangular pipe with an attenuation like that of the H_0 wave.

Nevertheless, a certain instability of the H_0 wave, predicted by Brillouin and caused by an almost unavoidable departure of the shape of the cross section of realizable pipes from that of an exact circle, does seem to take place. In a separate investigation of the transmission in pipes of elliptical cross section,[17] we have found that if a circular pipe becomes only slightly elliptical the curve of attenuation versus frequency for this wave will assume the general form (Fig. 24) that is typical of hollow-pipe waves. Therefore the attenuation will go through a minimum value and eventually increase with increasing frequency.

The expressions of attenuation constants for different types of waves in a pipe of circular cross section have been presented by Carson, Mead, and Schelkunoff[5] and for the E_0 wave by Barrow.[6] Making use of the nomenclature of this paper and rewriting these expressions in terms of a square pipe b centimeters on a side that has the same periphery as the circular pipe gives

[17] L. J. Chu, "Electromagnetic waves in elliptic hollow pipes of metal," *Jour. App. Phys.*, vol. 9, pp. 583–591; September, (1938).

E_0 wave:

$$\alpha = Kb^{-3/2}(0.862)\frac{\left(\frac{f}{f_0}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_0}\right)^{0}-1}}$$

E_1 wave:

$$\alpha = Kb^{-3/2}(1.087)\frac{\left(\frac{f}{f_0}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_0}\right)^{2}-1}}$$

H_0 wave:

$$\alpha = Kb^{-3/2}(1.087)\frac{\left(\frac{f}{f_0}\right)^{-1/2}}{\sqrt{\left(\frac{f}{f_0}\right)^{2}-1}} \qquad (43)$$

H_1 wave:

$$\alpha = Kb^{-3/2}(0.753)\frac{\left(\frac{f}{f_0}\right)^{-1/2}+\frac{1}{2.38}\left(\frac{f}{f_0}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_0}\right)^{2}-1}}$$

We cannot compare the minimum attenuation of an H_0 wave in a circular pipe with that of any wave in a rectangular pipe, because, theoretically at least, the attenuation of the H_0 wave can be reduced to an arbitrarily small value by the simple expedient of increasing the frequency indefinitely. However, for other types of waves, the attenuation in circular pipes passes through a minimum value. Table III shows the relative magnitudes of the minimum obtainable attenuations, the critical wavelengths, and the optimum wavelengths for waves in pipes of circular and square cross sections of equal peripheries.

TABLE III
FOR AIR-FILLED PIPES OF EQUAL PERIPHERIES

Shape of pipe	Wave type	$\frac{\alpha_{\min}/b^{3/2}}{\sqrt{\frac{2\pi\mu_1}{c\sigma_1\mu_1^2}}}$	$\frac{\lambda_c}{b}$	$\frac{\lambda_{opt}}{b}$
Circular	H_1	0.60	2.174	0.69
	E_0	1.20	1.662	0.96
	E_1	1.52	1.041	0.60
Square	$H_{0,1}$	1.12	2.000	0.68
	$H_{1,1}$	2.38	1.414	0.58
	$E_{1,1}$	1.92	1.414	0.82

If a source were available with a wavelength of $0.69b$, the optimum value for the H_1 wave, transmission could also be effected in the same pipe with the H_0 wave. However calculation shows that at this wavelength the attenuation of the H_0 wave is 42.5 per cent greater than that of the H_1 wave.

A study of these results leads to the following conclusions:

1. Any wave in a circular pipe has a smaller attenuation than does the corresponding wave in a square pipe of the same periphery.

2. The H_1 wave in a circular pipe has the smallest attenuation obtainable in any pipe if the wavelength is not smaller than $0.69b$.

Therefore, in the immediate future, and particularly when constructional aspects are to be considered, it would appear that the H_1 wave in a circular pipe is the most promising one for hollow-pipe transmission over long distances. On the other hand, the $H_{0,1}$ wave in a rectangular pipe will, because of its appropriate field pattern, probably find early application to radiation problems.

Proceedings of the Institute of Radio Engineers
Volume 26, Number 12 *December, 1938*

CHARACTERISTICS OF THE IONOSPHERE AT WASHINGTON, D. C., OCTOBER, 1938*

By
T. R. Gilliland, S. S. Kirby, and N. Smith
(National Bureau of Standards, Washington, D. C.)

DATA on the critical frequencies and virtual heights of the ionosphere layers during October are given in Fig. 1. Fig. 2 gives the maximum frequencies which could be used for sky-wave radio communication by way of the regular layers. As in September no

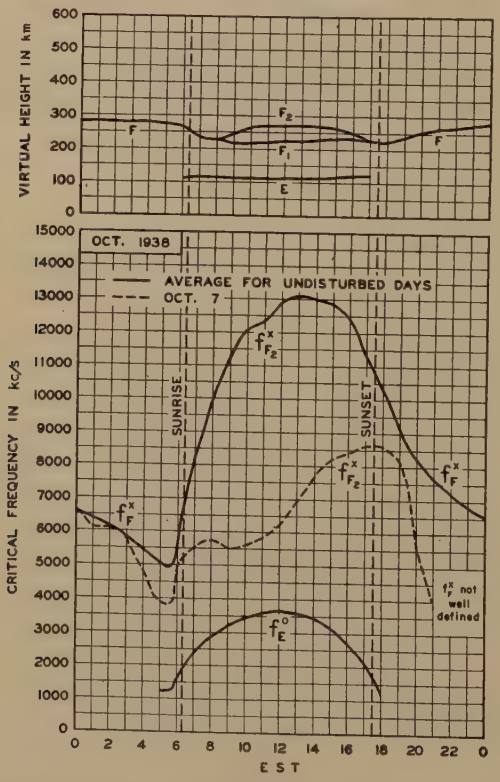

Fig. 1—Virtual heights and critical frequencies of the ionosphere layers, October, 1938.

* Decimal classification: R113.61. Original manuscript received by the Institute, November 9, 1938. These reports have appeared monthly in the Proceedings starting in vol. 25, September, (1937). See also vol. 25, pp. 823–840, July, (1937). Publication approved by the Director of the National Bureau of Standards of the U. S. Department of Commerce.

well-defined F_1-layer critical frequencies were observed, except during the ionosphere storm of October 7. The ionosphere storms and sudden ionosphere disturbances are listed in Tables I and II, respectively.

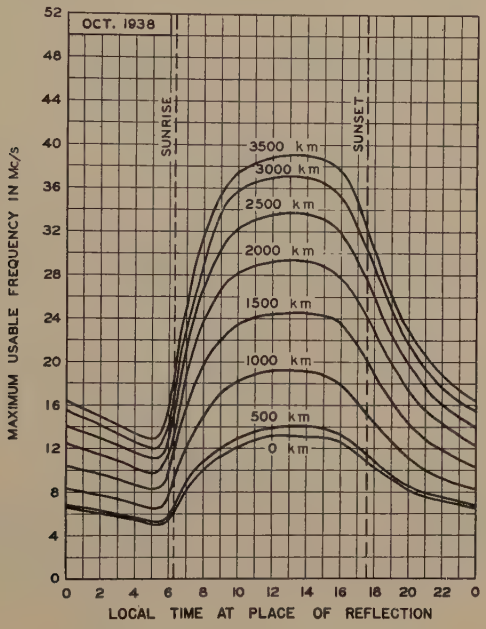

Fig. 2—Maximum usable frequencies for radio sky-wave transmission; averages for October, 1938, for undisturbed days, for dependable transmission by the regular F and F_2 layers.

TABLE I

IONOSPHERE STORMS (APPROXIMATELY IN ORDER OF SEVERITY)

Date and hour, E.S.T.	h_F before sunrise (km)	Minimum f^x_F before sunrise (kc)	Noon $f^x_{F_2}$ (kc)	Magnetic character[1]		Ionosphere character[2]
				00.12 G.M.T.	12–24 G.M.T.	
Oct.						
7 (after 0300)	377	3900	6100	0.8	1.4	1½
8	414	3500	about 10000	1.6	0.6	1
9 (until 0600)	318	4000	—	0.4	0.2	½
1 (after 2100)	—	—	—	1.3	0.6	½
2 (until 1700)	338	4500	10600	0.4	0.2	½
26	326	4200	11200	0.8	0.9	½
27 (until 0600)	342	4100	—	1.0	0.9	½
1 (until 0500)	392	4000	—	1.3	0.6	½
For comparison: Average for undisturbed days	281	4950	12900	0.2	0.3	0

[1] American magnetic character figure, based on observations of seven observatories.
[2] An estimate of the severity of the ionosphere storm at Washington on an arbitrary scale of 0 to 2, the character 2 representing the most severe disturbance.

TABLE II

SUDDEN IONOSPHERE DISTURBANCES

Date 1938	G.M.T. Beginning of fade-out	G.M.T. End	Location of transmitters recorded	Minimum relative intensity[1]
Oct. 6	1450	1510	Ohio, Ontario, Mass.	0.1
10	1928	1943	Ohio, Mass., D. C.	0.1
14	1540	1558	Ohio, Mass., D. C.	0.0
15	1857		Ohio, Mass., D. C.	0.0
15	1920	1940	Ohio, Mass.	0.0

[1] Ratio of received field intensity during fade-out to average field intensity before and after; for station W8XAL, 6060 kilocycles, 650 kilometers distant.

TABLE III

PERCENTAGE OF TIME FOR WHICH THE RATIO OF f_F^2 OR $f_{F_2}^2$ TO THE UNDISTURBED AVERAGE EXCEEDED THE RATIOS AT HEAD OF COLUMNS

1730 to 0830, E.S.T.

Ratio of critical frequency to average	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
Per cent of undisturbed time (369 hours)	100	100	100	100	92	50	8	0
Per cent of disturbed time (66 hours)	100	94	88	70	38	8	0	0
Per cent of total time (435 hours)	100	99	98	95	84	44	7	0

0830 to 1730, E.S.T.[1]

Per cent of undisturbed time (27 hours)	100	100	100	100	100	52	11	0
Per cent of disturbed time (9 hours)	100	100	100	78	44	11	0	0
Per cent of total time (36 hours)	100	100	100	94	86	42	8	0

[1] Data for one day a week only (Wednesday), because during this part of the day the F_1 critical frequencies exceeded the limit of recorders which operated every day.

TABLE IV

SPORADIC E

APPROXIMATE UPPER LIMITS OF FREQUENCY OF THE STRONGER SPORADIC-E REFLECTIONS AT VERTICAL INCIDENCE

Midnight to noon

Hour, E.S.T.

Date	00	01	02	03	04	05	06	07	08	09	10	11
Oct.												
12										8	8	4.5
18			4.5							8	8	4.5
19	6	8		8								
20				8	8	8	8			4.5	8	4.5
21	8	8									4.5	4.5

Noon to Midnight

Hour, E.S.T.

Date	12	13	14	15	16	17	18	19	20	21	22	23
Oct.												
18	4.5	4.5	4.5	4.5	4.5	4.5		8				
20					4.5		8	8	8			
22						8		8				
23							8	8		8		

In addition to the sudden ionosphere disturbances listed in Table II, there was a prolonged period of low-layer absorption, with received intensities falling slowly to zero and then rising again, from 1354 to 1700 G.M.T., October 16.

Data on the degree of departure during October of the F and F_2 critical frequencies from the averages in Figs. 1 and 2 are given in Table III. Information as to what time was disturbed is given in Table I.

The days during which sporadic E-layer reflections were most prevalent at Washington are listed in Table IV. The table shows the approximate upper limits of frequency at which strong sporadic E-layer reflections were observed at the hours listed. The observations were nearly continuous at 4.5, 6, and 8 megacycles.

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BOOK REVIEWS

American Standards for Electrical Indicating Instruments. Published by American Standards Association, 29 West 39th St., New York, N. Y. 20 pages. Price 40 cents.

Attention is directed to bulletin C-39 recently issued by the American Standards Association on "Standards for Electrical Indicating Instruments" which provides a common language for the various groups concerned with measurements by defining the several types of instruments, and the terms and expressions which are in general use. It specifies permissible temperature rise in instrument windings, shunts, external resistors, and multipliers; length of shunt leads; volt-ampere requirement for ammeters, voltmeters, wattmeters, etc., conditions under which instruments are tested with reference to voltage and overload; damping for different instruments; and limitations for the many "influences" which are now recognized. It is not implied that all of this is new as much work of this character has been done previously by the American Institute of Electrical Engineers but it does mean a wider acceptance of these standards.

This bulletin is a result of the combined efforts of representatives of eleven organizations serving on the A.S.A. Sectional Committee on Electrical Measuring Instruments. The Institute of Radio Engineers participated through representation by Mr. F. H. Drake.

*H. M. Turner

Engineering Electronics, by Donald G. Fink. McGraw-Hill Book Company, 330 West 42nd St., New York, N. Y. 292 pages plus 7-page index. Price $3.50.

"This book has been written to meet the needs of the practicing engineer who has a good foundation in electricity but who has no specific training in electronic concepts and methods," and also "for the student who wishes to orient himself in the field before undertaking advanced courses." The material selected and the presentation seem to be well adapted for such introductory purposes.

The material is divided into three sections, Physical Electronics, Electron Tubes, and Electron-Tube Applications. The first section covers the physics of free electrons in a vacuum, electron emission, and the gaseous discharge. The second section covers the various types of thermionic high-vacuum and gas-filled tubes, photosensitive tubes, electronic sources of light- and specialized electron tubes, such as cathode-ray and electron-multiplier tubes. The emphasis in this section is on methods of operation and operating characteristics rather than on tube structure or practical design. The third section contains chapters on elements of circuit theory, power-transformation circuits, communication circuits, and industrial-control circuits. In this section, the tube is considered only as a circuit element. Letter symbols and definitions are listed in appendixes.

* Yale University, New Haven, Conn.

In any moderate-sized book essaying to cover so wide a field, the majority of items must inevitably be treated very briefly and no reader will be entirely satisfied with the exact distribution of emphasis chosen by an author. This book will be subject to some criticism on this score but on the whole it is well balanced and includes a goodly amount of practical detailed information.

The practicing radio engineer will be occasionally annoyed by a too generalized statement or by the simplifications and half truths that are probably a necessary result of the galloping rate at which some of the subjects have to be covered. One important laboratory will be surprised to learn that the oxide emitter was neglected prior to 1920. An adequate bibliography is introduced at the end of each chapter with references to standard treatises covering the items touched on in this introductory survey. The author has wisely chosen to give only brief space to historical aspects. The book is up to date in its references to electronic devices and applications and quite inclusive.

Numerous useful tables and charts are provided. The problems at the end of each chapter may be helpful when the book is used as a text book. There is a good index. The organization of the material and the mechanical design are excellent.

*P. T. Weeks

The Low Voltage Cathode Ray Tube and Its Applications, by G. Parr (Ediswan Electric Co.) Chapman & Hall Ltd., London. 177 pages, 76 illustrations. Price 10s 6d.

In experimental laboratories the world over the cathode-ray oscillograph is rapidly becoming a useful, universal tool. This is especially, but not exclusively, true of the electrical and radio laboratories. Mr. Parr, the author, realizing that instruction in cathode-ray television technique is needed, has written this book. It covers the construction and operation of the tube itself, focusing, and performance. This is followed by a chapter on Lissajous' figures, something seldom used in modern oscillography today. In dealing with linear and other time bases all of the well-known circuits are shown.

The author briefly mentions a number of applications of the cathode-ray oscillograph to radio engineering, such as intermediate-frequency resonance curves, modulation measurements, phase-shift measurements, dynamic valve characteristics, power loss in dielectrics, observation of atmospherics, direction finding, and the like. Industrial applications under the following topics are also mentioned: resistance-pressure indicator, explosion meter, indicator diagrams, hysteresis loops, cathautograph, electrocardiograph.

The concluding chapter is on television reproduction. This appears to be the least interesting portion of the book, since none of the newer methods and circuits used for television are shown.

The appendix gives some notes on photography of the cathode-ray oscillograph trace and on television photography. A very good bibliography of 16 pages completes the book.

†A. F. Murray

* Raytheon Production Corporation, Newton, Mass.
† Philco Radio and Television Corporation, Philadelphia, Penna.

Electron Optics in Television, by Maloff and Epstein. Published by McGraw-Hill Book Company, 330 West 42nd St., New York, N. Y. 299 pages. Price $3.50.

The authors, two researchers of the RCA Manufacturing Company, have collected their writings on television subjects in this volume.

Television engineers will find that a large portion of the subject matter has been published previously in PROCEEDINGS I.R.E., *RCA Review*, *Electronics*, etc. Of course it is convenient to have these various papers collected between two covers, and this book will doubtlessly find itself in many television libraries.

Chapter 1, describing the RCA television system, is composed of extracts from RCA papers published during the past four years.

The portion of the book devoted to electron optics is of considerable interest to the electronic engineer who previously has found most of the publications on this subject in German; for instance, "Geometrische Elektronenoptik" by Brüche and Scherzer, 1934. Fundamental concepts are covered, followed by the usual statements regarding electron emission and an analogy between electron optics and light. As a background for the chapters to follow, motions of electrons in electrostatic fields are treated mathematically. This, as well as a discussion of electrostatic electron lenses, is in accord with usual accepted ideas.

Engineers find real practical help in the next chapter where the authors show calculations for electrostatic lenses in TCR (television cathode-ray) tubes.

Defects of the focusing system is informative and interesting to designers of TCR tubes who wish the authors had gone far enough to work out the gun tolerances permissible in order to produce a focused spot of satisfactorily small asymetrical aberration.

The magneto-static focusing section is rather brief. The inclusion of practical forms of magnetic lenses would be of value.

Electron guns, Chapter 9, is believed to be new material. It is filled with actual design data, appreciated by the workers in this branch of electronics.

Chapter 10 relates to deflection of electron beams. The following chapter describes luminescent screens, reporting some of the work done at RCA and elsewhere. Classifications and characteristics of TCR tubes are covered; data on projection tubes are included.

The title of Chapter 13 is "Accessories," meaning deflecting circuits or time bases. The material, largely mathematical, is not of as much interest to the TCR researcher as would have been a practical discussion of equipment, circuits, and the like.

The book is written in a very easy-to-read style, with commendably few errors. The illustrations are numerous and well-drawn.

*A. F. MURRAY

Telecommunications—Economics and Regulation, by James M. Herring and Gerald C. Gross. 544 pages (444 pages of text with 3 appendixes and index). McGraw-Hill Book Company, New York and London, 1936. Price $5.00.

"Telecommunication: Any telegraph or telephone communication of signs, signals, writings, images, and sounds of any nature, by wire, radio, or other systems or processes of electric or visual (semaphore) signalling."

* Philco Radio and Television Corporation, Philadelphia, Penna.

This paragraph, taken by the authors from the international treaty on communications now in effect, defines the somewhat unfamiliar term used as the principal title of the book. The book covers a wide field. A communication engineer is likely to find that while only certain chapters deal with his own specialty, the other chapters are very informative as to related matters acquaintance with which will broaden his view of the general subject.

Chapters I–IV are historical. They cover the development of the telegraph industry, submarine telegraphy, the telephone industry, and radio, respectively.

Chapters V–VIII deal with various aspects, particularly the economics, of present-day telephone and telegraph services—elements with which the engineer has increasing need to become familiar. In Chapter V the authors discuss the services from which the revenues of the communication industry are derived and the nature of the expenditures which are involved in the conduct of these services. In the case of the telegraph industry, for example, the authors emphasize the important relation of salaries and wages, particularly with respect to collection and delivery of messages, to the total operating costs. The discussion of radiotelegraph costs is brief; the authors point out that these facts are not a matter of public record. In the discussion of telephone service it is pointed out, among other things, that the expenditures for central offices (and to some extent for cables, poles, etc.) tend to increase faster than the number of subscribers, on account of the switching requirements. The authors emphasize that in the case of both telegraph service and telephone service operating costs are greatly affected by the load factor; i.e., the distribution of message traffic during the hours of the day.

Chapters IX to XIV relate to the history and practice of the regulatory process—state, national, and international. Chapter IX refers to the beginning of federal regulation in 1866 pursuant to certain provisions of the Post Roads Act. This chapter includes a discussion of the jurisdiction and activities of the Interstate Commerce Commission and the regulation of communicaton services and gives an unusually complete discussion of the authority of the President over the landing and operating of submarine cables conferred by the Submarine Cable Act approved May 27, 1921. Chapter X is a discussion of radio regulation beginning with the Ship Act of 1910 and the Radio Act of 1912; the subsequent Radio Act of 1927 necessitated by the advent of broadcasting is discussed in detail, particularly the Davis amendment relating to the establishment of quotas for broadcast facilities, which was enacted in 1928 and repealed in 1936. Chapter XI discusses in detail the regulation, under the Radio Act of 1927, of radio communication services other than broadcasting.

Chapter XV is a discussion of national policy with respect to communication services. The authors point out that while the Communications Act of 1934 is an expression of the view of Congress, that these services should be regulated in the public interest, the Act leaves undecided many of the longstanding controversial issues. The chapter continues with a discussion of a number of these issues, particularly those involved in broadcasting, such as the use of broadcasting for education as against advertising, the censorship of programs, and the duplication of programs on clear channels. It is interesting that some of the problems now actively under discussion are so new as not to be dealt with in this book, notably the question, as to whether newspapers should be permitted to own broadcast stations (certain statistics on newspaper ownership are given on page 107) and the problem of determining the price at which the sale of a broadcast station may be permitted.

In conclusion, the authors emphasize that while the Communications Act will require amendment, both the Commission and the Congress should recognize that regulation must be constructive as well as corrective.

Appendix A is a reprint of the Communications Act of 1934 (certain amendments, particularly with respect to ship radio matters, have been enacted by Congress since the printing of this book). Appendix B is a printing of the North and Central American Regional Radio Agreement, which resulted from the conference at Mexico City, July 10, 1933. Appendix C is a copy of certain provisions of the International Convention for the Safety of Life at Sea which was ratified by the U. S. Senate with certain reservations on June 19, 1936.

The book is replete with references to the sources from which the authors have obtained statistics and other facts. It is a valuable reference book for the worker in this field.

*Laurens E. Whittemore

Fernsehen, by Fritz Schröter. Published in German by Julius Springer, Berlin, 1937. 260 pages, 228 illustrations. Price 21 RM (cloth-bound).

The internationally known scientist, Dr. Schröter, has compiled in this book eight chapters each written by a different German television expert. The reader thus secures an excellent cross section of television development in Germany. It is doubtful if the American television engineer will find any outstandingly new material or ideas in this volume. However, it so completely covers the German television field that it is valuable as a marker—a milestone—of television progress up to 1937. Of course much has happened since then. Yearly editions should be published (with English translations!).

The eight sections are as follows:

Development and Status of the Television Art, by Dr. Banneitz.

This outlines a typical television system, shows the effect of varying the number of picture elements in a picture, mentions electron multipliers, delayed film transmission, ultra-short-wave transmitters and their range, the concentric cable, and the German telephone-television link.

The Physical Foundations, Possibilities and Limitations of Television Transmission, by Dr. Schröter.

Mentioned in this section are the following subjects: the photocell, glow lamps, light valves, scanning, resolution charts, and breadth-of-resonance curve.

Mechanical Scanning Systems and Synchronization, by Dr. Möller.

The latest German developments in mechanical scanning for film are described.

Geometrical Electron Optics, By Dr. Brüche.

We find mathematical and physical explanations of electron paths, lenses, focusing, the electron microscope, and electron optics in television (including a discussion of the types of American camera tubes).

The Cathode-Ray Tube in Television, by Dr. Knoll.

Both camera and picture tubes are discussed. Several photographs of good 375-line television pictures are shown. Readers interested in patent background

* American Telephone and Telegraph Company, New York, N. Y.

will find listed the early cathode-ray-tube workers: Braun (1897), Rosing (1907), Campbell Swinton (1908), George (1929). Regarding pickup tubes, there is shown on page 122, side by side, the Diekmann and Hell tube (1925) and the Farnsworth tube (1927). Concerning the Iconoscope type the author says: "The principle of storage of charges by means of multicellular arrangements was originated by R. Round (ref. 74) (1926). The scanning of such an arranagement by a cathode ray was first described by K. Tihanyi (ref. 75) (1928), the technical solution was made by Zworykin (ref. 76)."

Television Broadcasting, by W. Buschbeck.

Ultra-high-frequency transmitter problems are treated, including modulators, concentric cable, etc. Considerably more space than is justified is given to "Neutralization."

Television Reception, by M. von Ardenne.

Ultra-high-frequency field strength and noise level are mentioned. The picture receiver, portion by portion, is analyzed. Photographs of German receivers are shown.

The Large Picture Problem in Television, by Dr. Karolus.

The projection system which uses moving film between the picture tube and the projection screen is shown, also the multichannel system, the Kerr cell, the commutator method with a screen composed of many lamps, high-voltage cathode-ray projection tubes. The latter have been highly perfected in Germany.

The bibliography is a very valuable part of the book.

*A. F. Murray

* Philco Radio and Television Corporation, Philadelphia, Penna.

BOOKLETS, CATALOGS, AND PAMPHLETS RECEIVED

The following commercial publications of radio engineering interest have been received. Your request for a copy of any item may be addressed to the *Proceedings* for forwarding to the issuing company. Please mention your business affiliation.

AUDIO-FREQUENCY RELAYS • • • *Sigma Instruments, Inc. Data sheet No. 5, 1 page, 8½×11 inches, printed.* General description of 2 sensitive relays that respond to weak voice-frequency impulses.

CATHODE RAY TUBES • • • *Allen B. Du Mont Laboratories, Inc. Du Mont* Oscillographer *for August-September. 8 pages, 6×9¼ inches, printed.* Description of use of a cathode ray oscillograph in recording current-potential curves of a dropping mercury electrode (electrochemistry).

COAXIAL LINES • • • *Communication Products, Inc. Catalog Supplement No. 1, 4 pages, 8½×11 inches, lithograph.* Accessories for a ⅜-inch gas-filled coaxial line and a new receiving antenna feeder in which the internal dielectric is spun glass.

COMPONENTS • • • *The F. W. Sickles Company. Catalog No. 939, 24 pages, 8½×11 inches, lithograph.* Data on inductors, chokes, i-f transformers, condensers, and hardware.

COMMUNICATION RECEIVERS • • • *The National Company. Bulletin No. 280, 16 pages, 8½×11 inches, printed.* A catalog of communication-type receivers and parts.

CONDENSERS • • • *Cornell-Dubilier Electric Corporation. Catalog No. 161, 40 pages, 8½×11 inches, printed.* A general catalog giving specifications on condensers of the following types: dry and wet electrolytic, dykanol, mica, and paper.

DUMMY ANTENNA • • • *Ohmite Manufacturing Company. Bulletin III, 5½×8½ inches, lithographed.* Describes and gives performance data on a dummy antenna resistor mounted in an evacuated and gas-filled glass bulb.

HEADSETS • • • *Trimm Radio Manufacturing Company. Catalog, 6 pages+cover, 8½×11 inches, printed.* Lists a variety of headsets and ear phones.

METAL CABINETS • • • *Par-Metal Products Corporation. Catalog No. 39, 28 pages +cover, 8½×11 inches, lithograph.* Describes racks, panels, and cabinets in several finishes for commercial-transmitter service.

MICROPHONES • • • *Turner Company. Folder, 6 pages, 8½×11 inches, printed.* A listing of crystal and dynamic microphones in a variety of mountings.

PRECISION CONDENSERS • • • *General Radio Company.* Experimenter *for October–November, 12 pages, 6×9⅛ inches, printed.* Describes a new precision-type condenser with information for predicting its performance at frequencies up to 30 megacycles.

CONTRIBUTORS TO THIS ISSUE

Barrow, W. L.: Born October 25, 1903, at Baton Rouge, Louisiana. Received B.S. degree in electrical engineering, Louisiana State University, 1926; M.S. degree in electrical engineering, Massachusetts Institute of Technology, 1929; Sc.D. degree in physics, Technische Hochschule, Munich, Germany, 1931 (Redfield Proctor Fellow in Physics). Instructor and Round Hill Research, communications division, Massachusetts Institute of Technology, 1931–1936; assistant professor of electrical communications, 1936 to date. Member, American Institute of Electrical Engineers. Associate member, Institute of Radio Engineers, 1928.

Chu, Lan Jen: Born 1913 at Hwei-ying, Kiangsu, China. Received B.A. degree, Chiao-tung University, Shanghai, 1934; M.S. degree, Massachusetts Institute of Technology, 1935. Graduate student, Massachusetts Institute of Technology, 1935 to date. Member, Sigma Xi. Nonmember, Institute of Radio Engineers.

Gilliland, T. R.: See PROCEEDINGS for January, 1938.

Greene, F. M.: Born January 15, 1910, at Cedar Rapids, Iowa. Received B.S. degree in electrical engineering, Georgia School of Technology, 1933; M.S. degree in electrical engineering, Massachusetts Institute of Technology, 1937. Student engineer, WSB, Atlanta, Georgia, 1928–1933; engineering staff, 1933–1934; radio receiver engineering department, General Electric Company, summer, 1935; communications engineering, Erie Railroad Company, 1935 to date. Member, Committee on Research and Development, Association of American Railroads. Member, Tau Beta Pi, Phi Kappa Phi, and Kappa Eta Kappa. Associate member, Institute of Radio Engineers, 1935.

Kirby, S. S.: See PROCEEDINGS for January, 1938.

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Sinclair, D. B.: Born May 23, 1910, at Winnipeg, Manitoba, Canada. University of Manitoba, 1926–1929. Co-operative course in electrical engineering, Massachusetts Institute of Technology, 1929–1932; received S.B. degree in electrical engineering, 1931; S.M. degree, 1932; Sc.D. degree, 1935. Research assistant, Massachusetts Institute of Technology, 1932–1935; research associate, 1935–1936. Engineer, General Radio Company, 1936 to date. Member, Sigma Xi. Junior member, Institute of Radio Engineers, 1930; Associate, 1933; Member, 1938.

Smith, Newbern: See PROCEEDINGS for January, 1938.

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Pennsylvania	State College, 734 W. Foster Ave.	Crossley, G. L.
India	Calcutta, 9 Circus Ave.	Chakravarti, S. P.

Elected to the Associate Grade

Georgia	St. Simons Island, Box 71	Bryson, J. W.
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Kentucky	Lexington, University of Kentucky, Engineering Dept.	Doll, E. B.
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Canada	Toronto, Ont., 74 Edna Ave.	Hill, H. W.
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Egypt	Cairo, 59 Sharia Falaki	Thomas, J. P.
Holland	Amsterdam, da Costakade 19	Van Dyk, A.
India	Bombay 13, 3rd Laxmi Niwas, Elphinstone Rd.	Wakankar, V. W.
	Hyderabad (Dn.), 510 Sultan Bazar	Joshi, D. P.
New Zealand	Auckland, 65 Brooklyn Flats	Sloane, G. B.

Elected to the Junior Grade

Illinois	Chicago 2118 W. Evergreen Ave.	Sarnowicz, H. V.
Oregon	Portland, KOIN, New Heathman Hotel	Price, D. A.
England	Erdington, Birmingham 24, 3 Arthur Rd.	Yerbury, G. A.
India	Lucknow, 29 Kaisarbagh	Saksena, V. K.

Elected to the Student Grade

California	Berkeley, 576 Santa Barbara Rd.	Berkley, J. B.
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Massachusetts	Cambridge, M.I.T. Dormitories	Roberts, S.
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APPLICATIONS FOR MEMBERSHIP

Applications for transfer or election to the various grades of membership have been received from the persons listed below and have been approved by the Admissions Committee. Members objecting to transfer or election of any of these applicants should communicate with the Secretary on or before December 31, 1938. These applications will be considered by the Board of Directors at its meeting on January 4, 1939.

For Transfer to the Member Grade

Georgia	Atlanta, 1479 Lanier Pl., N.E.	Fowler, N. B.
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	South Orange, 9 Crest Circle	Poppele, J. R.
England	Leigh-on-Sea, Essex, Little Birches, Woodside	Robertson, N. C.
India	Karachi, N. E. D. Engineering College	Row, D. P.

For Election to the Member Grade

Germany	Berlin, Olympische Strasse 2	Knoll, M.
South Australia	Adelaide, "Burnbrae," Hartland Ave., Black Forest	Hall, L. S.

For Election to the Associate Grade

California	Arcadia, 1011 S. Mayflower Ave.	Jones, J. G.
	Long Beach, 2632 Van Buren St.	Corcoran, M. E.
	Modesto, P. O. Box 2	Bates, W. H., Jr.
District of Columbia	Washington, 3009 Ordway St., N.W.	Brustmann, J.
	Washington, 4709 Overbrook Rd., Friendship Station	Stuart, D. M.
Florida	Miami, 9454 N.E. 4th Ave.	Stuhrman, A. P.
Illinois	Chicago, 3503 W. Flournoy St.	Dacken, L.
	Chicago, c/o United Air Lines, 5936 S. Cicero Ave.	Jensen, E. A.
	Elmwood Park, 2711-76th Court	Hansen, E. S.
Massachusetts	Fall River, 48 Almy St.	Oliver, N. J.
Missouri	Kansas City, 1601 Broadway St.	Carapezza, S.
Nebraska	Omaha, 3323 Harney St.	Grant, D. S.
New Jersey	Harrison, RCA Manufacturing Company, Inc.	Glover, A. M.
New York	Brooklyn, 365 Quincy St.	Happe, W. H., Jr.
	New York, Bell Telephone Labs, Inc., 463 West St.	Fair, I. E.
	New York, 2251 Loring Pl.	Grumich, J.
	New York, Ferranti Electric, Inc., 30 Rockefeller Plaza	Spittal, W. R.
Pennsylvania	Pittsburgh, Gulf Research and Development Co., Box 2038	Thomas, E. U.
Texas	San Antonio, c/o KTSA, Gunter Hotel	Egerton, W. G.
Wisconsin	Burlington, 623 Briody St.	Anderson, L. C.
Australia	South Brisbane, Queensland, 12 Glenelg St.	Ham, G.
British West Indies	Kingston, Jamaica, c/o Jamaica Public Service	Metcalf, E.
Canada	New Toronto, Ont., 263-7th St.	Leslie, I. M.
England	Dollis Hill, London N.W. 2, P. O. Research Station Radio Section	Simmonds, J. C.
	Islington N.1, London, 26 Douglas Rd.	Grenly, M.
Japan	Saitamaken, Tokio Daini Hosojo, Hatogayamachi	Imaeda, R.
Southern Rhodesia	Salisbury, Box 3	Moss, A. J.

For Election to the Junior Grade

Straits Settlements	Singapore, c/o The British Malaya Broadcasting Corp., Ltd.	Kai Boh, K.

For Election to the Student Grade

California	Berkeley, 818 Spruce St.	Graham, R. B.
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	Berkeley, U. S. Naval R.O.T.C. Unit	Lee, L. K.
	Oakland, 284-9th St.	Pettit, J. M.
	Palo Alto, 710 Hanover St.	MacQuivey, D. R.
	Stanford University, Box 1651	LeMasters, W. G.
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	Angola, 201 E. Broad St.	Taylor, R. R.
	Angola, 408 S. Oakwood St.	Thomas, F. G.
	Lafayette, St. Elizabeth Hospital	Jacobs, L. F.
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	West Lafayette, Purdue Union Bldg., Rm. 211	Trebby, F. J.
Massachusetts	Cambridge, M.I.T. Dormitories	Alexander, R. M.
	Cambridge, 74 Oxford	Sledge, O. D.
	Cambridge, 290 Massachusetts Ave.	Smullin, L. D.
Oregon	Corvallis, 2021 Western Ave.	Walker, E.
	Corvallis, 2021 Western Ave.	Walker, R. T.
Washington	Seattle, 4528-5th Ave., N.E.	Harrold, W. T.
Wisconsin	Madison, 728 W. Johnson St.	Kurth, H. H.
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I certify that the statements made in the record of my training and professional experience are correct, and agree if elected, that I will be governed by the constitution of the Institute as long as I continue a member. Furthermore I agree to promote the objects of the Institute so far as shall be in my power, and if my membership shall be discontinued will return my membership badge.

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(Title and name of concern)

Business Address .....

Permanent Home Address .....

Place of Birth ..... Date of Birth ..... Age .....

Education .....

Degree .....
(College) (Date received)

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DATES	

Record may be continued on other sheets of this size if space is insufficient.

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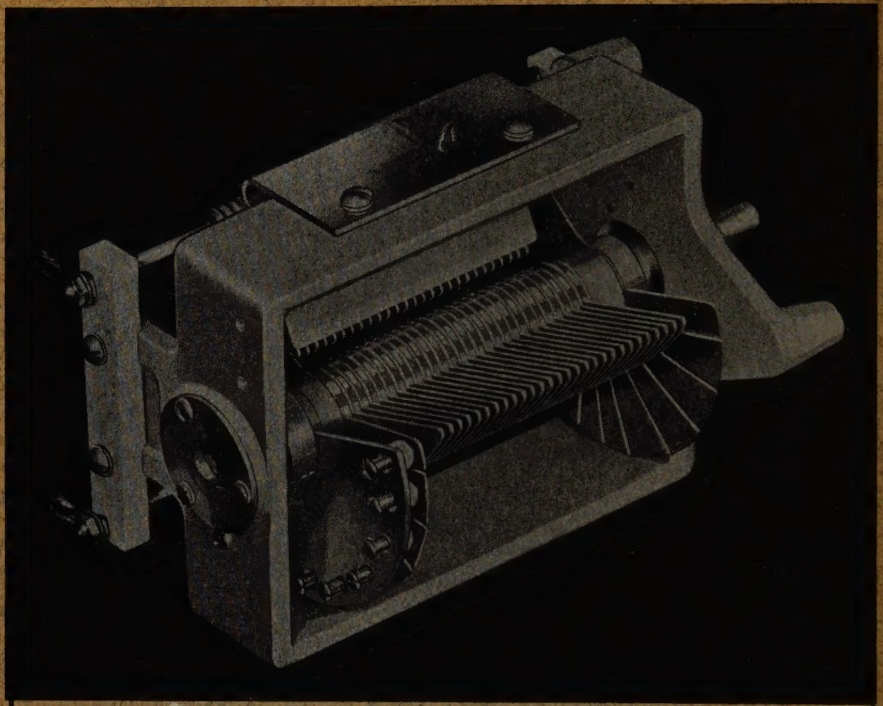

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GENERAL RADIO COMPANY

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